

# An MILP Approach for Short-term Scheduling of Batch Operations

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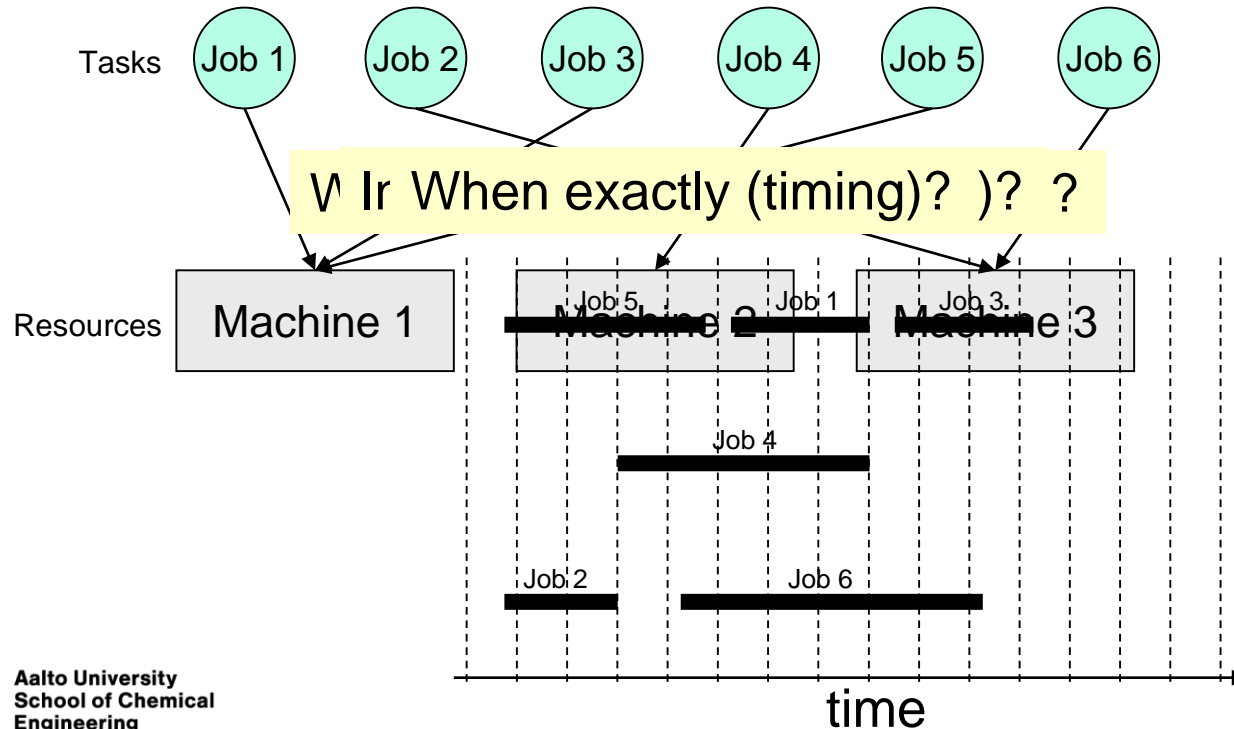
Date: Wednesday, 19<sup>th</sup> June 2019



Aalto University  
School of Chemical  
Engineering



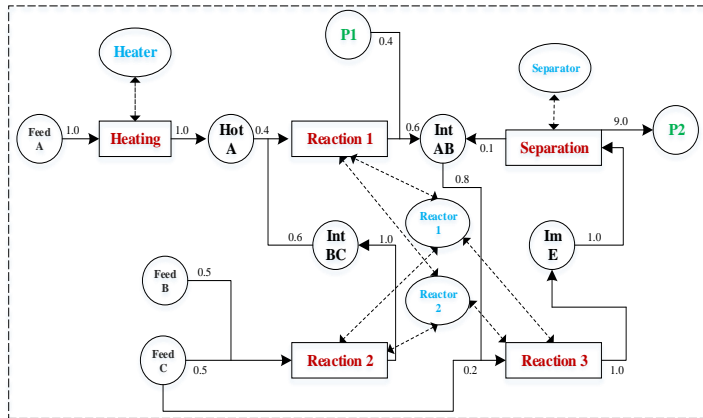
# Scheduling → 3 Key-decisions



# Introduction

## Batch Plant Scheduling

- Optimal allocation of a set of limited resources to some tasks over time
- Generic representations of batch process: Resource-Task Network (Pantelides, 1994) and State-task Network (Kondili et al., 1993)

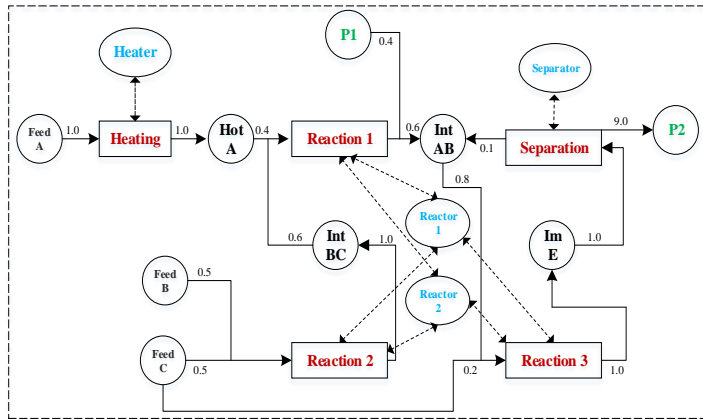


Resource-Task Network

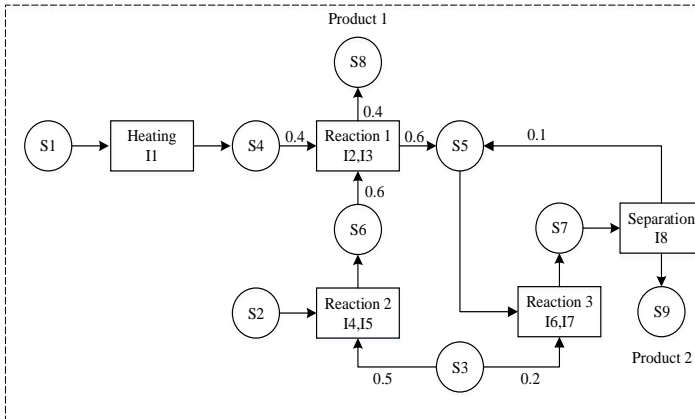
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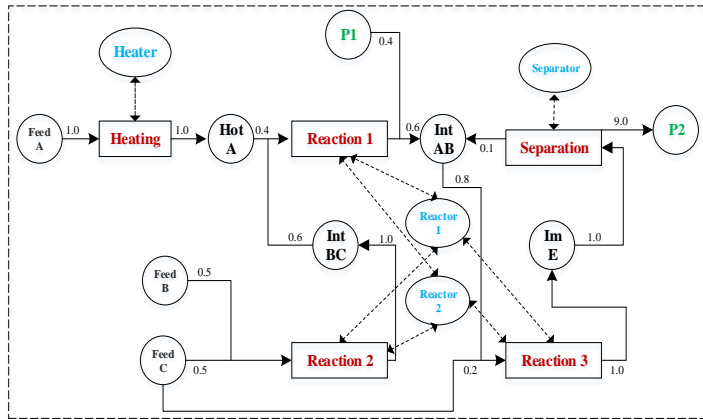


State-Task Network

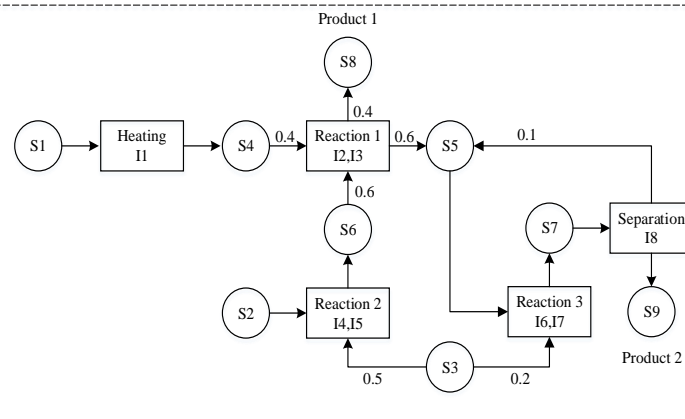
# Introduction

## Batch Plant Scheduling

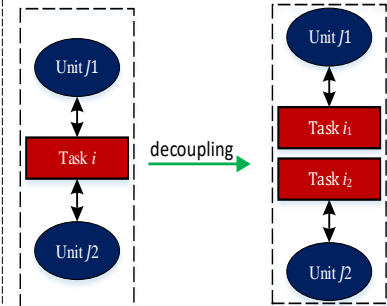
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Resource-Task Network

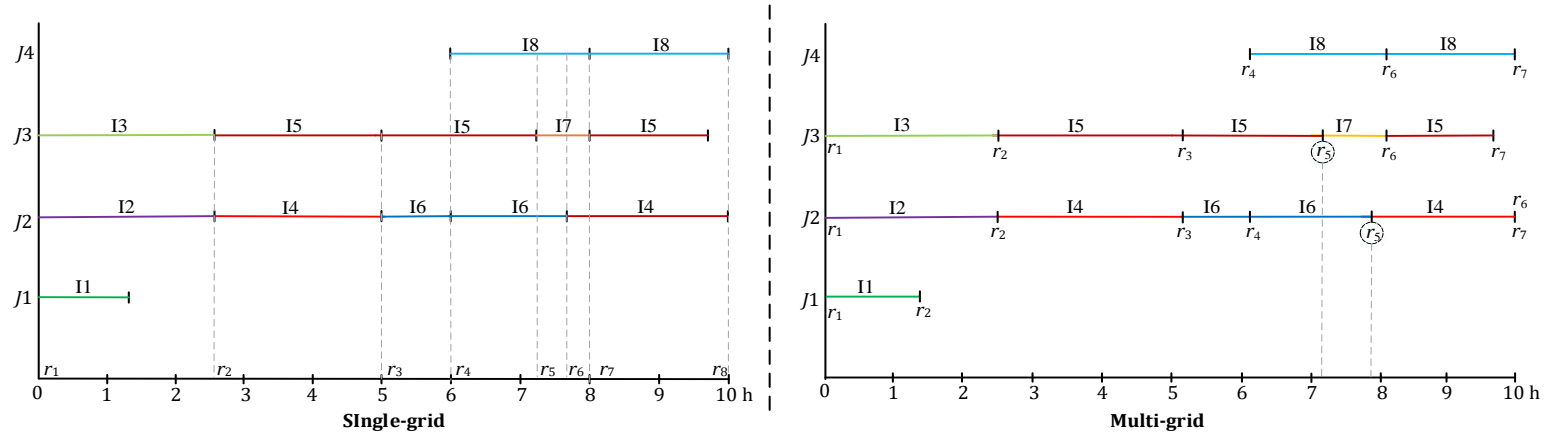


State-Task Network



# Time representation

- **Common reference grid (Single-grid, SG)**
  - The time slots are common for all units
- **Non-common reference grid (Multiple-grid, MG)**
  - The occurrences of each event can vary across the units



# Common constraints for SG and MG

## Batch size and processing time

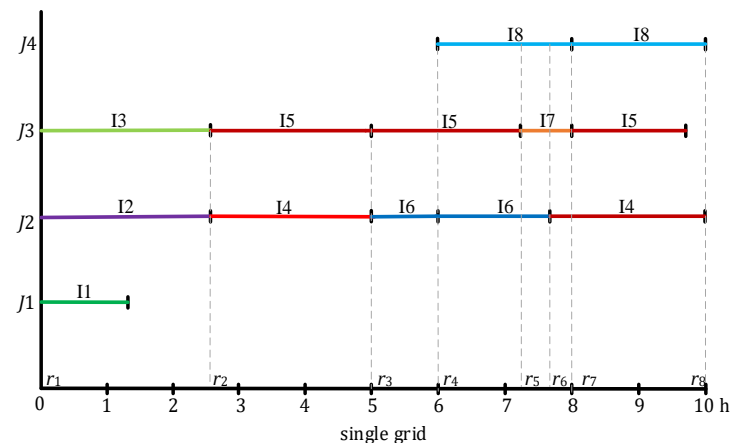
- $X_{i,r,r'} = 1$  task  $i$  is processed during time interval  $[r, r'] |_{r < r' \leq \Delta r + r}$
- $V_{i,r,r'} =$  batch size of task  $i$  during time interval  $[r, r'] |_{r < r' \leq \Delta r + r}$
- $LR_{i,r,r'} =$  processing time of task  $i$  during time interval  $[r, r'] |_{r < r' \leq \Delta r + r}$

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$$\bigvee_{i \in I_j} \left[ \begin{array}{c} X_{i,r,r'} \\ v_i^{\min} \leq V_{i,r,r'} \leq v_i^{\max} \\ LR_{i,r,r'} = cp_i + vp_i V_{i,r,r'} \end{array} \right] \bigvee \left[ \begin{array}{c} X_{r,r'}^{\text{no task}} \\ V_{i,r,r'} = 0, \quad \forall i \in I_j \\ LR_{i,r,r'} = 0, \quad \forall i \in I_j \end{array} \right] \quad \forall j \in J, r, r' | r < r' \leq r + \Delta r$$



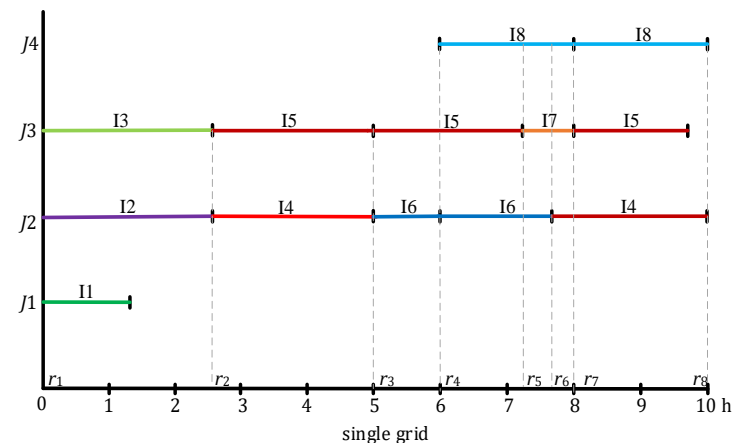


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$$\sum_{i \in I_j} X_{i,r,r'} \leq 1, \quad \forall j \in J, r, r' |_{r < r' \leq r + \Delta r}$$

$$v_i^{\min} X_{i,r,r'} \leq V_{i,r,r'} \leq v_i^{\max} X_{i,r,r'} \quad \forall i \in I_j, j \in J, r, r' |_{r < r' \leq r + \Delta r}$$

$$LR_{i,r,r'} = cp_i X_{i,r,r'} + vp_i V_{i,r,r'} \quad \forall i \in I_j, j \in J, r, r' |_{r < r' \leq r + \Delta r}$$

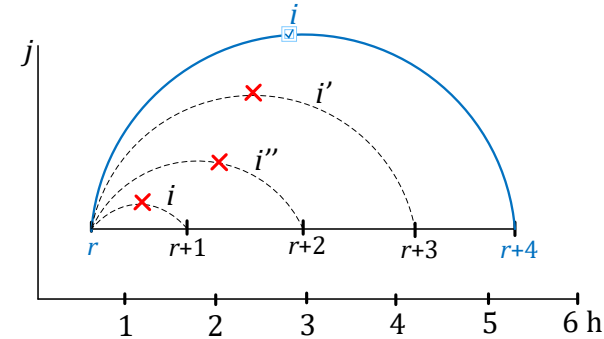
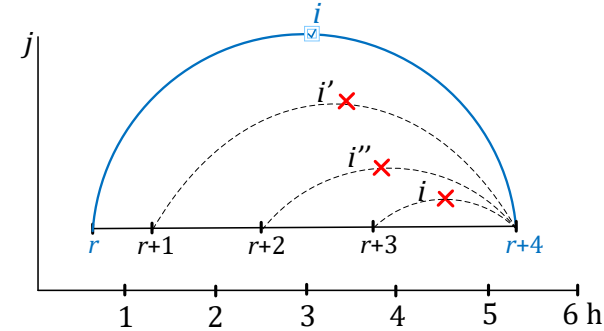
# Common constraints for SG and MG

## Allocation constraints

- For each unit only one task can start and finish at each time point

$$\bigvee_{i \in I_j} \bigvee_{\substack{r' \in R \\ r < r' \leq r + \Delta r}} X_{i,r,r'} \bigvee_{-} X_{r,r'}^{no\ i} \quad \forall j \in J, r \in R$$

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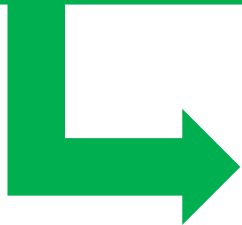
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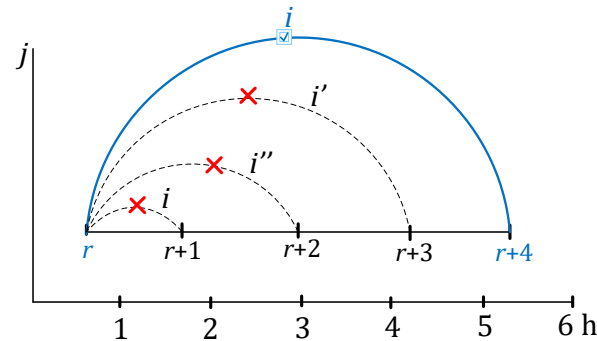
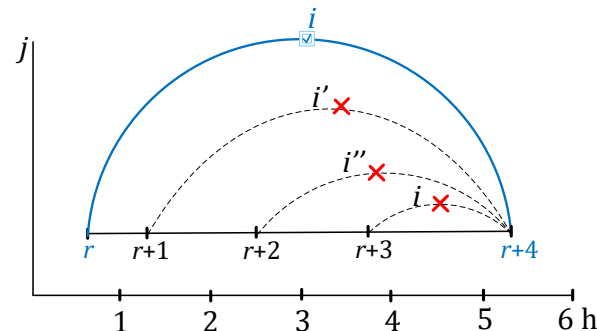
$$\bigvee_{i \in I_j} \bigvee_{\substack{r' \in R \\ r < r' \leq r + \Delta r}} X_{i,r,r'} \bigvee_{-} X_{r,r'}^{no\ i} \quad \forall j \in J, r \in R$$

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$$\sum_{r \in R} \sum_{\substack{i \in I_j \\ r < r' \leq r + \Delta r}} X_{i,r,r'} \leq 1 \quad \forall j \in J, r' \in R$$

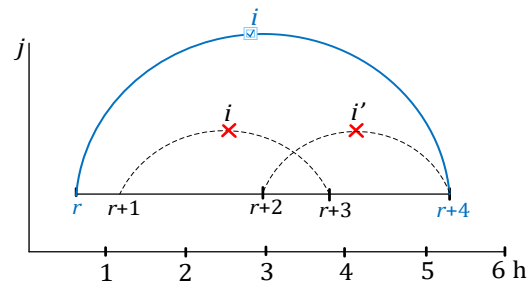
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# Common constraints for SG and MG

## Allocation constraints

- Task  $i$  is processed in unit  $j$  during time interval  $[r, r'] |_{r+1 < r' \leq r+\Delta r}$ , the same or other tasks suitable in unit  $j$  cannot be processed in any time interval  $[k, r''] \subseteq [r, r']$



$$X_{i,r,r'} \Rightarrow \neg \left( \bigvee_{\substack{r'' \in R \\ k < r'' \leq k+\Delta r}} X_{i',k,r''} \right) \quad \forall i, i' \in I_j, r, r', k \in R |_{r+1 < r' \leq r+\Delta r, r+1 \leq k, r' \geq k+1}$$



$$X_{i,r,r'} + \sum_{k < r'' \leq k+\Delta r} X_{i',k,r''} \leq 1 \quad \forall i, i' \in I_j, r, r', k \in R |_{r+1 < r' \leq r+\Delta r, r+1 \leq k, r' \geq k+1}$$

# Common constraints for SG and MG

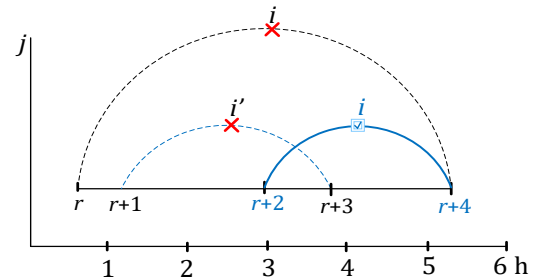
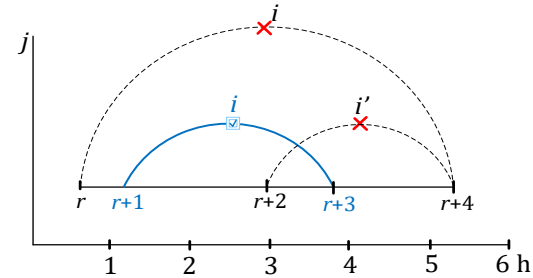
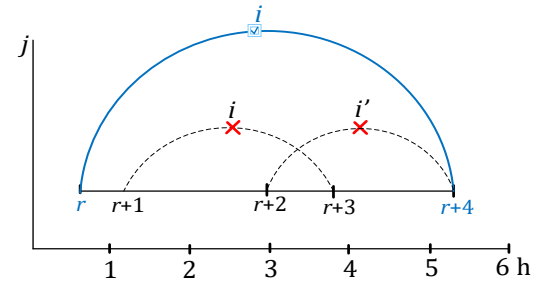
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# Common constraints for SG and MG

## Allocation constraints

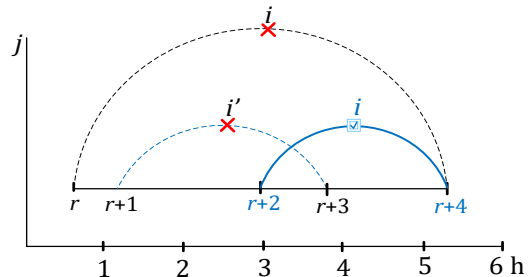
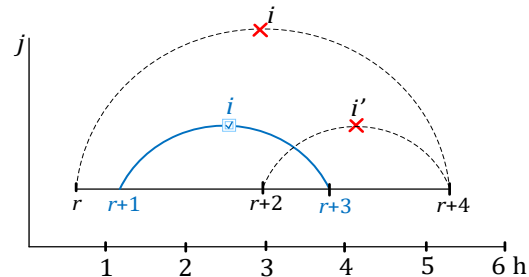
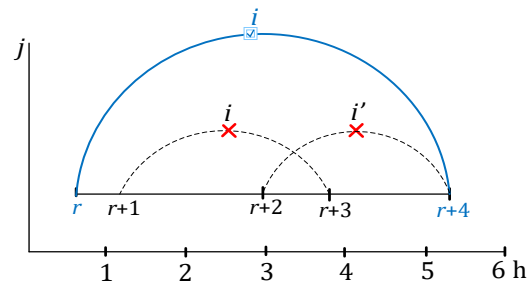
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$$X_{i,r,r'} + \sum_{k < r'' \leq k+\Delta r} X_{i',k,r''} \leq 1 \quad \forall i, i' \in I_j, r, r', k \in R |_{r+1 < r' \leq r+\Delta r, r+1 \leq k, r' \geq k+1}$$

$$(P \Rightarrow Q) \equiv (\neg Q \Rightarrow \neg P)$$



# Common constraints for SG and MG

- **Mass balance**

$F_{s,r}$  excess amount of state  $s$  at time point  $r$

$$F_{s,r} = f_s^0|_{r=1} + F_{s,r-1}|_{r>1} + \sum_{i \in I_s^p} \rho_{i,s}^p \sum_{\substack{r' \in R \\ r' < r \leq r' + \Delta r}} V_{i,r',r} + \sum_{i \in I_s^c} \rho_{i,s}^c \sum_{\substack{r' \in R \\ r < r' \leq r + \Delta r}} V_{i,r,r'} \quad \forall s \in S, r \in R$$

- **Meeting demand**

Demands ( $d_s$ ) at states  $s \in SM$  storing final products are enforced as a hard at

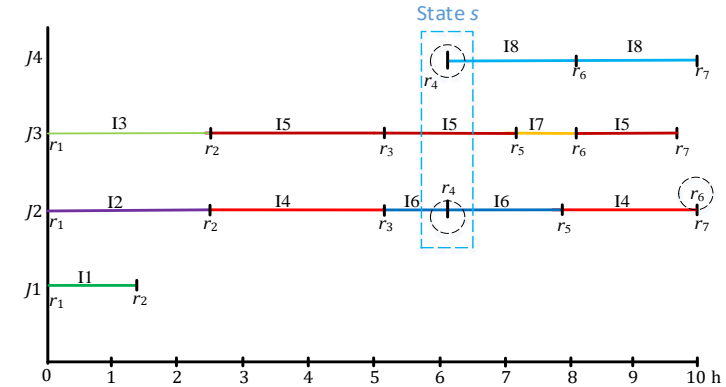
$$F_{s,r} \geq d_s, \quad \forall s \in SM, r = |R|$$

# Timing constraints

- **Timing constraints for MG**

➤  $SR_{j,r}$  the time that unit  $j$  starts at event point  $r$

$$\begin{aligned}
 SR_{j,r'} &\geq SR_{j,r} + \sum_{i \in I_j} LR_{i,r,r'} \quad \forall j \in J, r, r' | r < r' \leq r + \Delta r \\
 SR_{j,r'} &\geq SR_{j',r} + \sum_{i \in I_{j'}} LR_{i,r,r'} \quad \forall j \in J_s^c, j' \in J_s^p (j \neq j'), s \in S, r, r' | r < r' \leq r + \Delta r \\
 SR_{j,r} + \sum_{i \in I_j} LR_{i,r,r'} &\leq h_{\max} \quad \forall j \in J, r, r' \in R | r < r' \leq r + \Delta r
 \end{aligned}$$





# Timing constraints

- Timing constraints for MG

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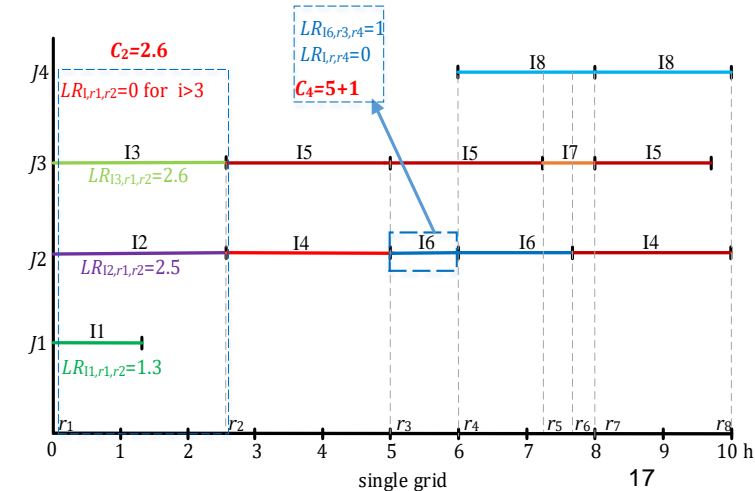
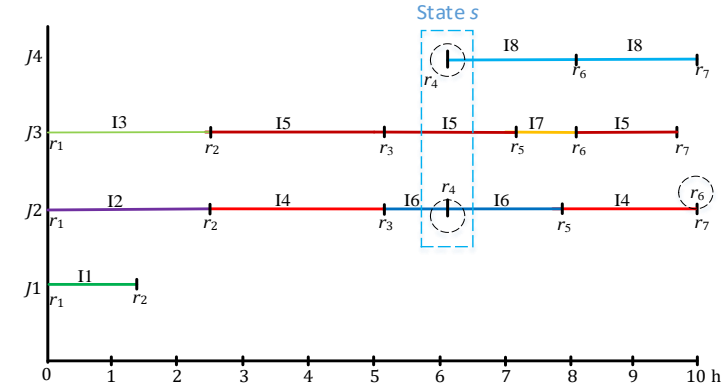
$$SR_{j,r} + \sum_{i \in I_j} LR_{i,r,r'} \leq h_{\max} \quad \forall j \in J, r, r' \in R | r < r' \leq r + \Delta r$$

- Timing constraints for SG

- $CR_r$  the absolute time of event point  $r$

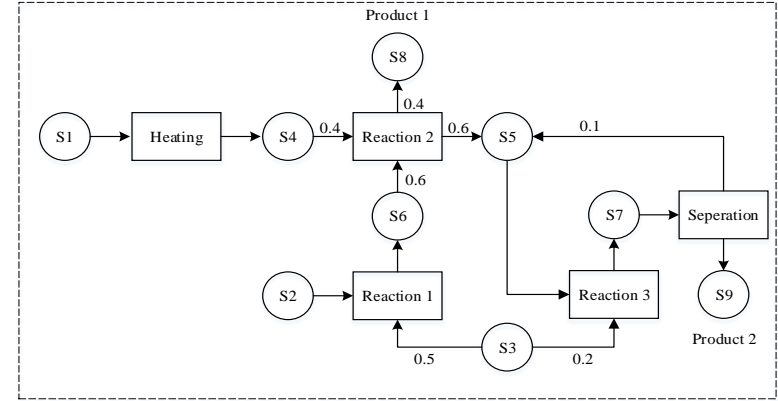
$$CR_{r'} - CR_r \geq LR_{i,r,r'} \quad \forall i \in I_j, j \in J, r, r' \in R | r < r' \leq r + \Delta r$$

$$C_r \leq h_{\max} \quad \forall r \in R$$

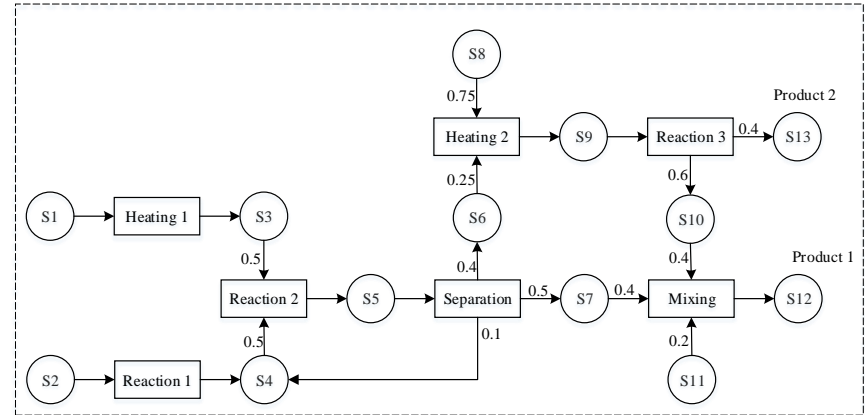


# Results and discussion

- **Two benchmark problems Ex1-Ex2**
  - Maravelias and Grossmann 2003 (M&G) vs. SG
  - Shaik and Floudas 2009 (S&F) vs. MG
- **GAMS 24.9.1 / CPLEX 12.7.1**
  - Option optcr =  $10^{-3}$
  - Option threads = 0
  - Option reslim = 7200 s.



State-task network representation for Ex 1



State-task network representation for Ex 2

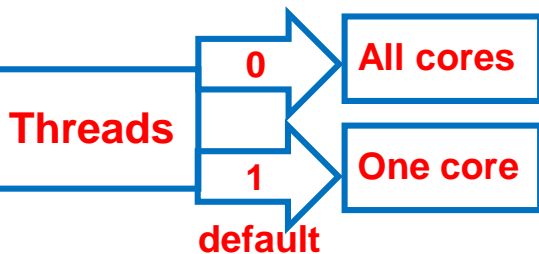
# Results and discussion

## Effect of threads on the CPU time and solution quality

- Cost maximization
- Time horizon 10 h
- Global optimum = 2358.2

Profit maximization

Optcr	S&F (8 event points , $\Delta n = 1$ )			MG (9 event points , $\Delta r = 2$ )		
	CPU (s)	MILP (\$)	RMILP (\$)	CPU (s)	MILP (\$)	RMILP (\$)
<b>Option threads = 0</b>						
10 <sup>-1</sup>	10.96	2338.7	3618.6	9.35	2331.3	3618.6
10 <sup>-2</sup>	549.5	2345.3	3618.6	102.7	2345.3	3618.6
10 <sup>-3</sup>	353.7	2358.2	3618.6	119.6	2358.2	3618.6
10 <sup>-4</sup>	462.4	2358.2	3618.6	146.6	2358.2	3618.6
10 <sup>-5</sup>	462.7	2358.2	3618.6	145.7	2358.2	3618.6
10 <sup>-6</sup>	459.1	2358.2	3618.6	147.3	2358.2	3618.6
<b>Option threads = 1</b>						
10 <sup>-1</sup>	31.9	2292.5	3618.6	51.65	2330.9	3618.6
10 <sup>-2</sup>	572.6	2345.3	3618.6	655.1	2358.2	3618.6
10 <sup>-3</sup>	875.4	2358.2	3618.6	947.6	2358.2	3618.6
10 <sup>-4</sup>	714.8	2358.2	3618.6	979.4	2358.2	3618.6
10 <sup>-5</sup>	710.7	2358.2	3618.6	950.5	2358.2	3618.6
10 <sup>-6</sup>	708.4	2358.2	3618.6	950.0	2358.2	3618.6



Changing threads option from 0 to 1 increases the CPU time considerably!

Number of cores = 4

# Results and discussion

- Effect of threads on the CPU time and solution quality

- Cost maximization
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- Global optimum = 2358.2**

Optcr	Profit maximization					
	S&F (8 event points , $\Delta n = 1$ )			MG (9 event points , $\Delta r = 2$ )		
	CPU (s)	MILP (\$)	RMILP (\$)	CPU (s)	MILP (\$)	RMILP (\$)
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Changing threads option also affect the  
Solution quality!

# Results and discussion (SG vs. M&G)

## Maximum revenue

- Same number of event points
- Both perform equally in 3 cases
- SG performs better in 2b (10 times faster)
- Fewer constraints, but slightly more binary variables
- SG leads to weaker RMILP

	Event points	CPU (s)	Binary variables	Total variables	Eqs.	MILP (\$)	RMILP (\$)
Ex 1a ( $H = 8 h$ )							
M&G	5	0.23	80	496	1095	1498.56	1730.8
SG	5 ( $\Delta r = 1$ )	0.04	32	147	212	1498.56	1730.8
	5 ( $\Delta r = 2$ )	0.06	56	219	368	1498.56	1730.8
Ex 1b ( $H = 10 h$ )							
M&G	8	25.01	128	793	1719	1962.69	2690.5
SG	8 ( $\Delta r = 1$ )	0.17	56	249	359	1860.72	2775.4
	8 ( $\Delta r = 2$ )	6.50	104	393	671	1958.99	2775.6
	8 ( $\Delta r = 3$ )	9.45	144	513	1031	1962.69	2775.6
	8 ( $\Delta r = 4$ )	10.87	176	609	1399	1962.69	2775.6
Ex 2a ( $H = 8 h$ )							
M&G	7	7.50	154	946	2076	1583.44	2560.6
SG	7 ( $\Delta r = 1$ )	0.50	66	297	436	1274.48	2750.9
	7 ( $\Delta r = 2$ )	0.70	121	462	781	1583.44	2750.9
	7 ( $\Delta r = 3$ )	0.76	165	594	1157	1583.44	2750.9
Ex 2b ( $H = 10 h$ )							
M&G	10	7200 <sup>a</sup>	220	1351	2934	2307.66	3473.9
SG	10 ( $\Delta r = 1$ )	5.87	99	438	643	1963.88	3618.6
	10 ( $\Delta r = 2$ )	326.92	187	702	1195	2156.36	3618.6
	10 ( $\Delta r = 3$ )	148.4	264	933	1853	2307.66	3618.6
	10 ( $\Delta r = 4$ )	188.29	330	1131	2567	2307.66	3618.6

RMILP

<sup>a</sup>Relative gap (RG)=8.67%  
700 s

# Results and discussion (SG vs. M&G)

## Maximum revenue

- Same number of event points
- Both perform equally in 3 cases (out of 6)
- SG performs better in other 3 cases
- Fewer constraints, but slightly more binary variables
- Roughly same RMILP

	Event points	CPU (s)	Binary variables	Total variables	Eqs.	MILP (h)	RMILP (h)
		Ex 1c ( $d_{SG} = 200$ mu, $d_{S9} = 200$ mu)					
M&G	10	7.06	160	992	2137	19.34	18.68
SG	10 ( $\Delta n = 1$ )	8.65	72	318	459	19.78	18.68
	10 ( $\Delta n = 2$ )	1.39	136	510	875	19.34	18.68
	10 ( $\Delta n = 3$ )	1.79	192	678	1379	19.34	18.68
		Ex 1e ( $d_{SG} = 600$ mu, $d_{S9} = 600$ mu)					
M&G	20	7200 <sup>a</sup>	320	1982	4217	46.52	45.57
SG	20 ( $\Delta n = 1$ )	7200 <sup>b</sup>	152	658	949	46.52	45.57
		Ex 1d ( $d_{SG} = 500$ mu, $d_{S9} = 400$ mu)					
M&G	25	7200 <sup>c</sup>	400	2477	5257	56.81	56.05
SG	25 ( $\Delta n = 1$ )	7200 <sup>c</sup>	192	828	1194	56.81	56.05
		Ex 2c ( $d_{S12} = 100$ mu, $d_{S13} = 200$ mu)					
M&G	11	71.1	242	1487	2980	13.36	11.33
SG	11 ( $\Delta n = 1$ )	0.62	110	486	714	14.61	11.25
	11 ( $\Delta n = 2$ )	6.09	209	783	1335	13.53	11.25
	11 ( $\Delta n = 3$ )	11.07	297	1048	2087	13.36	11.25
	11 ( $\Delta n = 4$ )	13.03	374	1278	2920	13.36	11.25
		Ex 2d ( $d_{S12} = 250$ mu, $d_{S13} = 250$ mu)					
M&G	12	23.60	264	1622	3508	17.02	14.40
SG	12 ( $\Delta n = 1$ )	0.79	121	533	783	18.97	14.27
	12 ( $\Delta n = 1$ )	1.36	231	863	1473	17.02	14.27
	12 ( $\Delta n = 3$ )	1.78	330	1160	2319	17.02	14.27
		Ex 2e ( $d_{S12} = 930$ mu, $d_{S13} = 840$ mu)					
M&G	29	691.4	638	3917	8370	51.82	50.92
SG	29 ( $\Delta n = 1$ )	12.43	308	1332	1956	59.74	49.92
	29 ( $\Delta n = 2$ )	49.28	605	2223	3819	51.82	49.92
	29 ( $\Delta n = 3$ )	110.21	891	3081	6263	51.82	49.92

<sup>a</sup>RG=2.01%, <sup>b</sup>RG=0.79%, <sup>c</sup>RG=1.15%

# Results and discussion (MG vs. S&F)

## Maximum revenue

- Both perform equally well (2 cases out 4)
- MG performs better than SF 1b and 2b
- MG leads to tighter formulations
- MG needs on more event point
- But, same binary and fewer Eqs

	Events	CPU (s)	Binary variables	Total variables	Eqs.	MILP (\$)	RMILP (\$)
Ex 1a ( $H = 8$ h)							
S&F	4 ( $\Delta n = 0$ )	0.14	32	165	307	1498.56	1730.8
MG	5 ( $\Delta r = 1$ )	0.10	32	162	250	1498.56	1730.8
Ex 1b ( $H = 10$ h)							
S&F	8 ( $\Delta n = 1$ )	1829.87	120	441	1255	1962.69	2805.4
MG	9 ( $\Delta r = 2$ )	302.12	120	478	920	1962.69	2804.2
Ex 2a ( $H = 8$ h)							
S&F	6 ( $\Delta n = 1$ )	8.82	121	453	1280	1583.44	2750.9
MG	7 ( $\Delta r = 2$ )	3.73	121	497	977	1583.44	2682.0
Ex 2b ( $H = 10$ h)							
S&F	8 ( $\Delta n = 2$ )	1570.12	231	743	2008	2358.20	3618.6
MG	9 ( $\Delta n = 3$ )	321.20	231	865	1997	2358.20	3618.64

# Results and discussion (MG vs. S&F)

## Minimum makespan

- Both perform equally well (5 cases out 6)
- MG performs better than S&F in Ex 2e (22.89 s vs. 7200)
- MG leads to tighter formulations
- MG needs on more event points
- But, same binary and fewer Eqs

	Events	CPU (s)	Binary variables	Total variables	Eqs.	MILP (h)	RMILP (h)
Ex 1c ( $d_{S8} = 200$ mu, $d_{S9} = 200$ mu)							
S&F	9 ( $\Delta n = 0$ )	1.10	72	371	652	19.34	18.68
MG	10 ( $\Delta n = 1$ )	1.09	72	348	547	19.34	18.68
Ex 1d ( $d_{S8} = 500$ mu, $d_{S9} = 400$ mu)							
S&F	20 ( $\Delta n = 0$ )	0.59	160	822	1477	46.11	45.57
MG	21 ( $\Delta n = 1$ )	1.48	160	755	1196	46.11	45.57
Ex 1e ( $d_{S8} = 600$ mu, $d_{S9} = 600$ mu)							
S&F	25 ( $\Delta n = 0$ )	3.17	200	1027	1852	56.68	56.05
MG	26 ( $\Delta n = 1$ )	2.25	200	940	1491	56.68	56.05
Ex 2c ( $d_{S12} = 100$ mu, $d_{S13} = 200$ mu)							
S&F	7 ( $\Delta n = 0$ )	0.21	77	401	702	13.36	11.25
MG	8 ( $\Delta n = 1$ )	0.34	77	385	631	13.36	12.31
Ex 2d ( $d_{S12} = 250$ mu, $d_{S13} = 250$ mu)							
S&F	10 ( $\Delta n = 0$ )	0.46	110	572	1017	17.02	14.27
MG	11 ( $\Delta n = 1$ )	0.45	110	541	892	17.02	14.53
Ex 2e ( $d_{S12} = 930$ mu, $d_{S13} = 840$ mu)							
S&F	29 ( $\Delta n = 0$ )	1013	319	1655	3012	51.82	49.92
	29 ( $\Delta n = 1$ )	7200 <sup>a</sup>	627	2271	6299	51.82	49.92
MG	30 ( $\Delta n = 1$ )	5.35	319	1529	2545	51.82	50.26
	30 ( $\Delta n = 2$ )	22.89	627	2453	4981	51.82	49.92

<sup>a</sup>Relative GAP=3.68%

22.89 s vs. 7200 s



# Conclusions

## Novel continuous-time MILP scheduling formulation for multipurpose batch plants

- **Pros**
  - Little effort is required to move from a single-grid model to a multi-grid one
  - No big-M constraints
  - The formulation can be extended to address continuous processes (Grade change, crude oil pooling problem, pipeline scheduling)
  - Tighter LP-relaxation with MG
- **Cons**
  - Two tuning parameters ( $r$  and  $\Delta r$ )
  - Weaker LP-relaxation with SG
  - The single grid model is not capable of handling changeover times constraints

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**Thank you for your attention!**

**Any Questions?**