Aalto University
School of Chemical
Engineering

Single reference grid continuous-time formulation for batch scheduling

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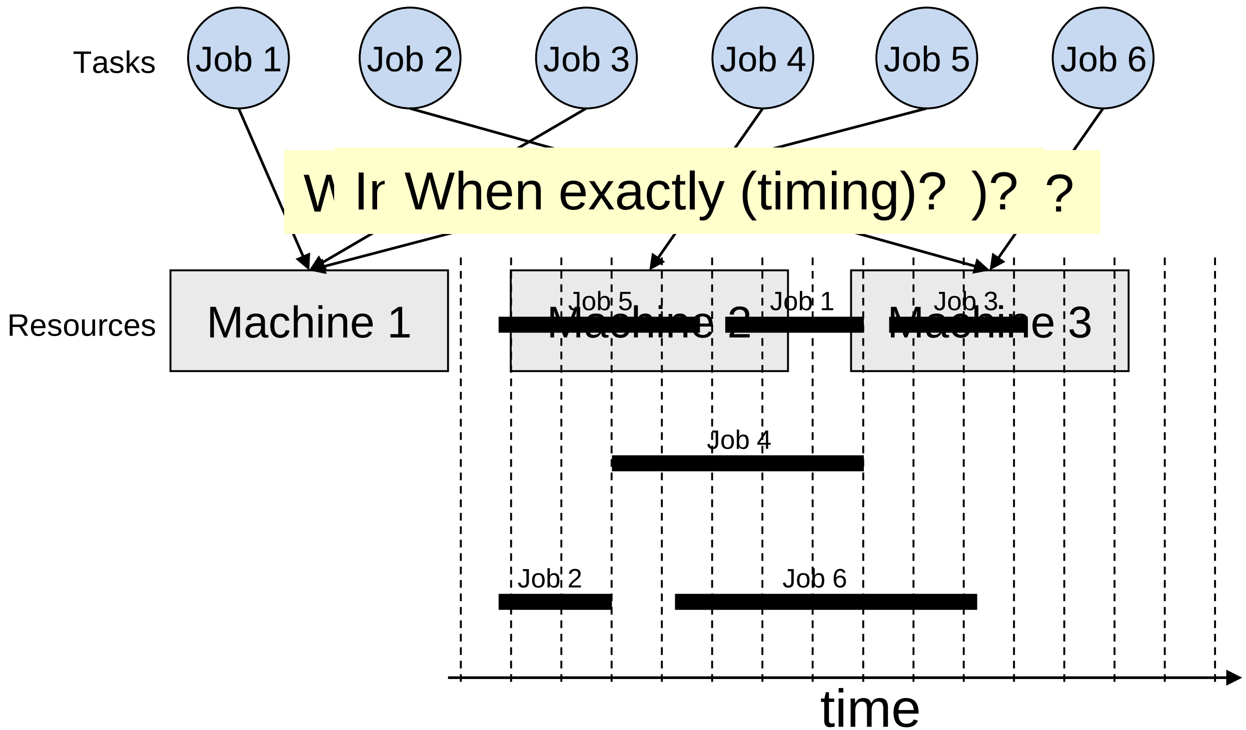
IFAC
INTERNATIONAL FEDERATION
OF AUTOMATIC CONTROL

DYCOPS 2019
12th IFAC Symposium on Dynamics and Control of
Process Systems, including Biosystems

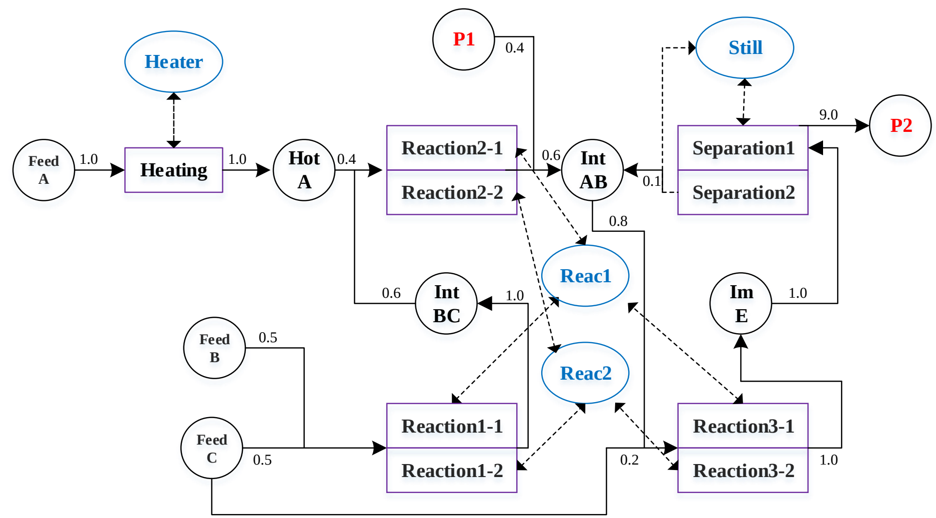
Scheduling → 3 Key-decisions



Introduction

- **Batch Plant Scheduling**

- Optimal allocation of a set of limited resources to some tasks over time
- Generic representations of batch process: State-task Network (Kondili et al., 1993) and Resource-Task Network (Pantelides, 1994)



Resource-task network for a plant producing two final products

Introduction

- **Given**

- Production recipe (processing time and amount of material)
- The available units and their capacity limits
- The available storage capacity for the materials
- The time horizon length and demand

- **To be determined**

- Optimal sequence of tasks in units (sequencing)
- Amount of material processed in each unit (batch size)
- Start and end time of each task in each unit (timing)

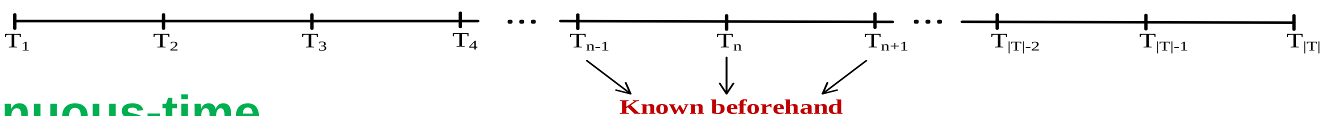
- **Objectives**

- Maximum profit, minimum cost or makespan

Time representation

- **Discrete-time**

The length of time slots is known beforehand (from minute to hours)



- **Continuous-time**

Time representation

- ## Discrete-time

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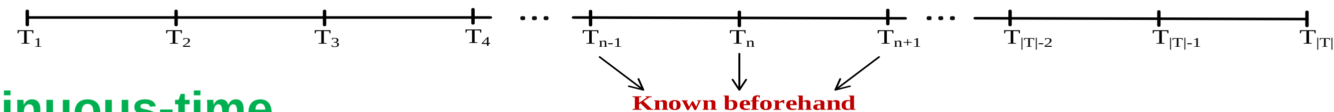
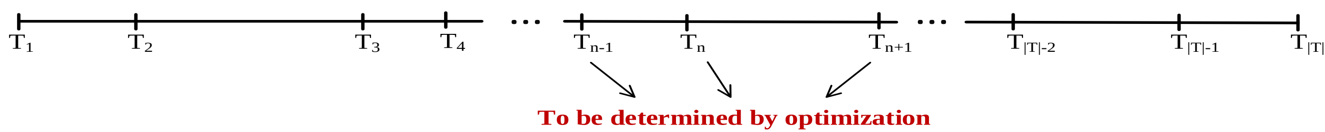


- ## Continuous-time

The length of time slots is a continuous variable



Time representation

singpro

- Discrete-time

pros

- Variable processing times
- Fewer number of time slots
- Smaller problem size

- Continuous-time

Time representation

- **Discrete-time**
- **Continuous-time**

pros

- **Variable processing times**
- **Fewer number of time slots**
- **Smaller problem size**

cons

- **Poor LP-relaxation**
- **Minimum number of time slots is unkown**

Scheduling formulation

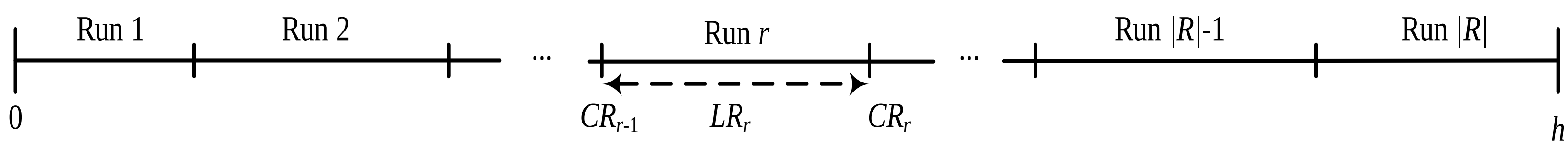
- We use STN representation to model the problem
- To arrange tasks in units we introduce the concept of run
  - Run =time slots
  - Place holder for a task
- First MILP model for batch plant fully derived from Generalized Disjunctive Programming (GDP) followed by a convex hull reformulation

Scheduling formulation

- **Sequencing of runs**
  - **Run r always starts after run r-1**

CR_r: Completion time of run r

LR_r: Duration of run r



Scheduling formulation

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CR_r: Completion time of run r

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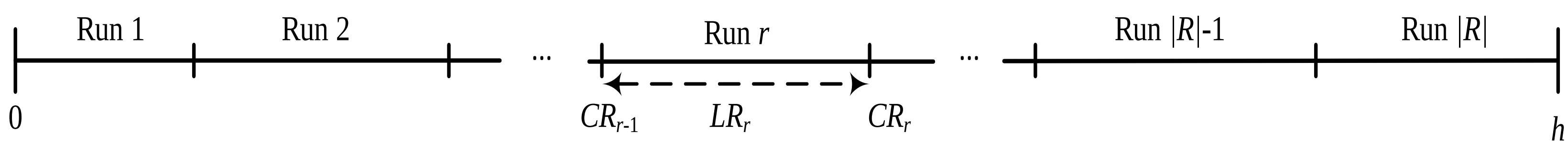
$$CR_r - LR_r \geq CR_{r-1}, \qquad \forall r|_{r\geq 1}$$

$$\sum_{r\in R} LR_r \leq H$$



Scheduling formulation

- ## Allocation and processing time

  - **At most one task in a unit per run**

$X_{i,j,r}$ = 1 if task i is processed in unit j during run r

$X_{j,r}^{no\ i}$ = 1 if unit j is idle during run r

$V_{i,j,r}$ = Batch size of task i in unit j during run r

$$\bigvee_{i \in I_j} \begin{bmatrix} X_{i,j,r} \\ v_{i,j}^{\min} \leq V_{i,j,r} \leq v_{i,j}^{\max} \\ LR_{j,r} \geq cp_{i,j} + vp_{i,j} V_{j,r} \end{bmatrix} \bigvee \begin{bmatrix} X_{j,r}^{no\ i} \\ V_{i,j,r} = 0, \quad \forall i \in I_j \\ LR_{j,r} = 0 \end{bmatrix}, \forall j, r$$

GDP framework

Scheduling formulation

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GDP framework

Logical relation	Comments	Boolean Expression	Representation as Linear Inequalities
Logical OR		$Y_1 \vee Y_2 \vee \ldots \vee Y_n$	$y_1+y_2+\ldots+y_n \geq 1$
Logical AND		$Y_1 \wedge Y_2 \wedge \ldots \wedge Y_n$	$\left\{ \begin{array}{l} y_1=1 \\ y_2=1 \\ \ldots \\ y_n=1 \end{array} \right.$
Implication	$Y_1 \Rightarrow Y_2$	$\neg Y_1 \vee Y_2$	$1-y_1+y_2 \geq 1$
Equivalence	Y_1 if and only if Y_2 ($Y_1 \Rightarrow Y_2 \wedge Y_2 \Rightarrow Y_1$)	$(\neg Y_1 \vee Y_2) \wedge (\neg Y_2 \vee Y_1)$	$y_1=y_2$
Exclusive OR	Exactly one of the variables is true	$Y_1 \underline{\vee} Y_1 \underline{\vee} \ldots \underline{\vee} Y_n$	$y_1+y_2+\ldots+y_n=1$

Scheduling formulation

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Exclusive OR	Exactly one of the variables is true	$Y_1 \veebar Y_1 \veebar ... \veebar Y_n$	$y_1+y_2+...+y_n=1$

$$\bigvee_{i \in I_j} \begin{bmatrix} X_{i,j,r} \\ v_{i,j}^{\min} \leq V_{i,j,r} \leq v_{i,j}^{\max} \\ LR_{j,r} \geq cp_{i,j} + vp_{i,j} V_{i,j,r} \end{bmatrix} \bigvee \begin{bmatrix} X_{j,r}^{no\ i} \\ V_{i,j,r} = 0, \quad \forall i \in I_j \\ LR_{j,r} = 0 \end{bmatrix}, \forall j, r$$

GDP framework

Convex hull

$$\sum_{i \in I_j} X_{i,j,r} + X_{j,r}^{no\ i} = 1 \Rightarrow \sum_{i \in I_j} X_{i,j,r} \leq 1 \quad \forall j, r$$

$$v_{i,j}^{\min} X_{i,j,r} \leq V_{i,j,r} \leq X_{i,j,r} \quad \forall i \in I_j, j, r$$

$$LR_{j,r} \geq \sum_{i \in I_j} cp_{i,j} X_{i,j,r} + \sum_{i \in I_j} vp_{i,j} V_{i,j,r} \quad \forall j, r$$

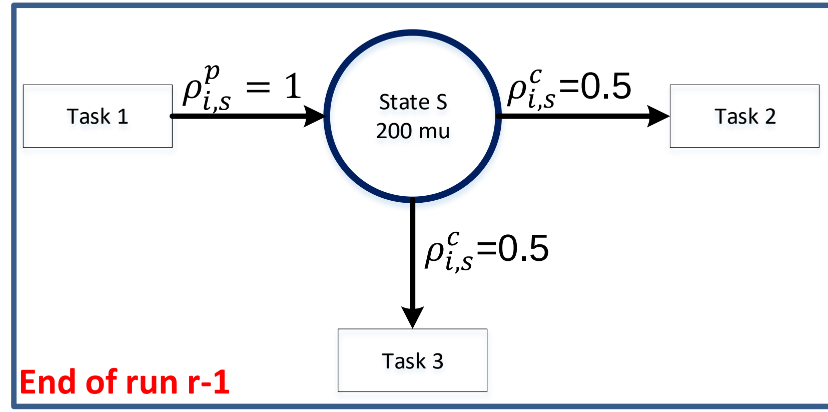
MILP formulation

Scheduling formulation

- **Mass balance**

$$ST_{s,r} = ST_{s,r-1} + \sum_{i \in I_s^p} \rho_{i,s}^p \sum_{j \in J_i} V_{i,j,r} - \sum_{i \in I_s^c} \rho_{i,s}^c \sum_{j \in J_i} V_{i,j,r} \quad \forall s, r|_{r \geq 1}$$

$$\sum_{i \in I_s^c} \rho_{i,s}^c \sum_{j \in J_i} V_{i,j,r} \leq ST_{s,r-1} \quad \forall s, r|_{r \geq 1}$$



Scheduling formulation

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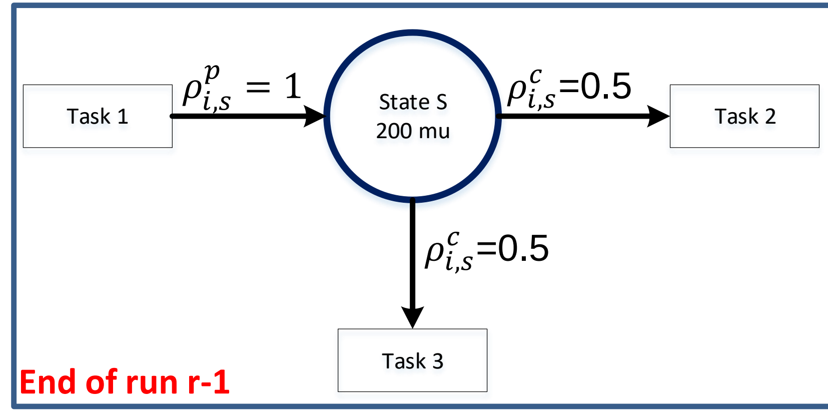

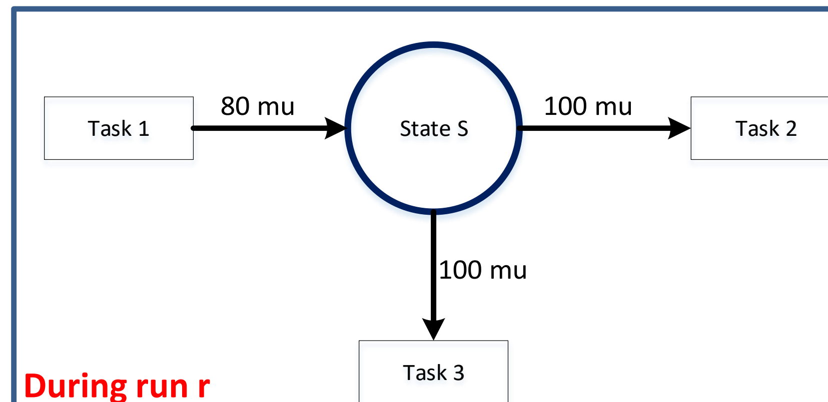

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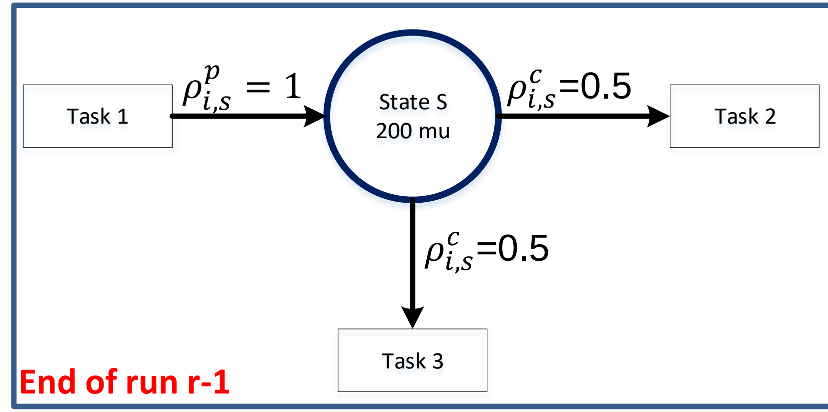
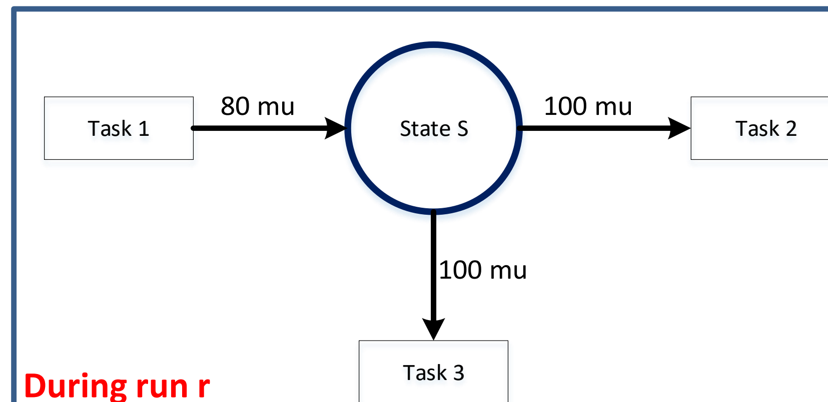
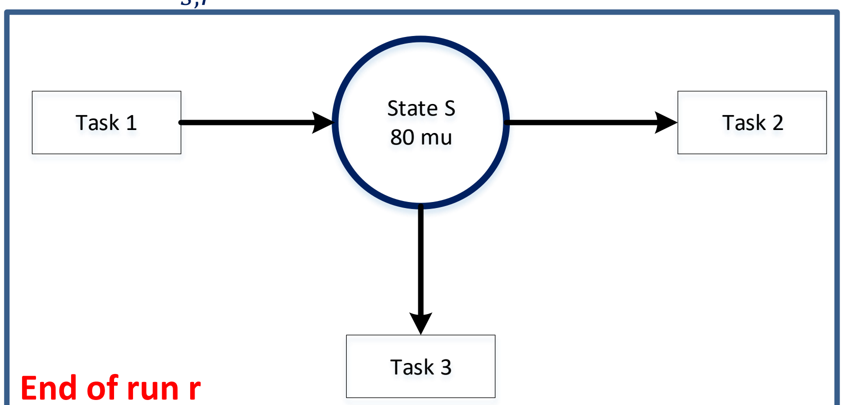
$$\sum_{i \in I_s^c} \rho_{i,s}^c \sum_{j \in J_i} V_{i,j,r} \leq ST_{s,r-1} \quad \forall s, r|_{r \geq 1}$$

$$ST_{s,r} = 200 + 80 - 100 - 100 = 80$$





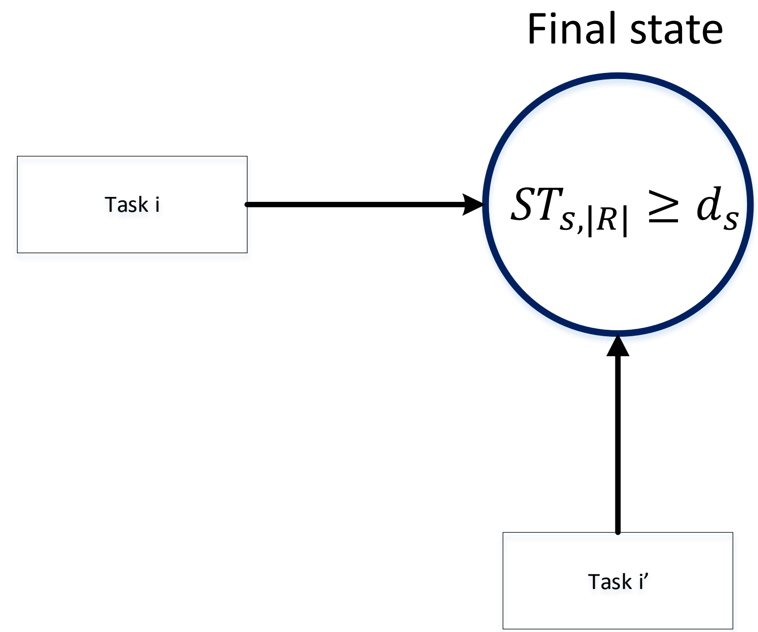


Scheduling formulation

- **Meeting demand**

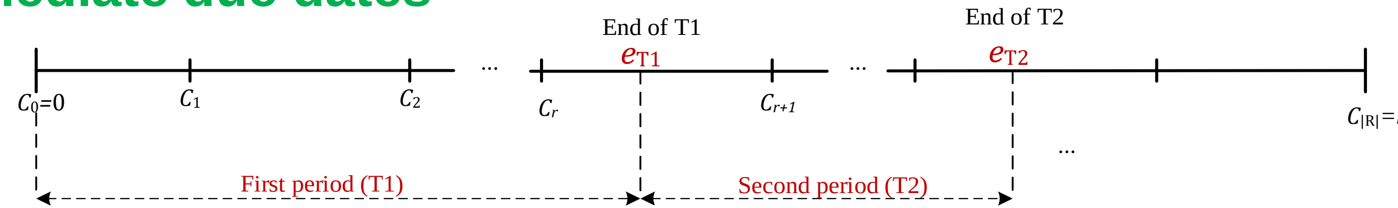
$$ST_{s,r} \geq d_s \quad \forall s, r|_{r\geq|R|}$$

Demand for state S, known parameter



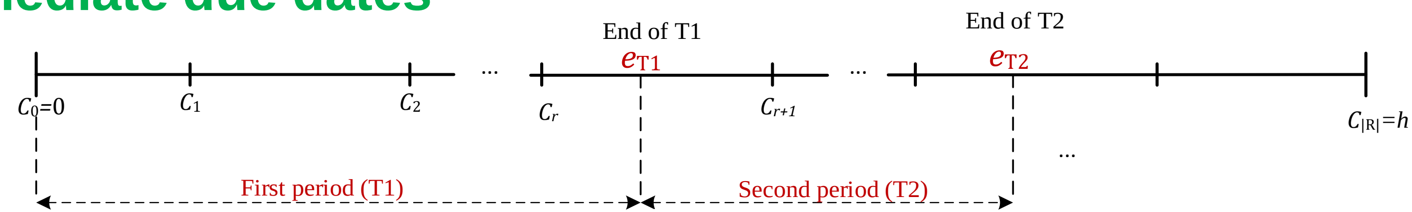
Scheduling formulation

- **Intermediate due dates**



Scheduling formulation

- ## Intermediate due dates



Each run should finish in exactly one period

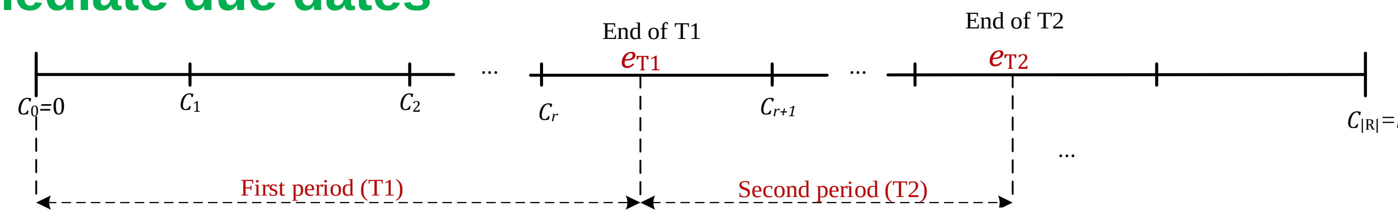
$$\boxed{\bigvee_t \begin{bmatrix} Y_{r,t} \\ e_{t-1} \le C_r \le e_t \end{bmatrix}, \quad \forall r}$$

$$\sum_t Y_{r,t} = 1, \quad \forall r$$

$$\sum_t Y_{r,t} e_{t-1} \le C_r \le \sum_t Y_{r,t} e_t \quad \forall r$$

Scheduling formulation

- ## Intermediate due dates



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$$\sum_t Y_{r,t} = 1, \quad \forall r$$

$$\sum_t Y_{r,t} e_{t-1} \le C_r \le \sum_t Y_{r,t} e_t \quad \forall r$$

Demand in each period should be satisfied on time

$$Y^{\text{last}}_{R,\text{T1}} = 1 \Rightarrow ST_{s,R} \ge d_{s,\text{T1}}$$

$$Y^{\text{last}}_{R',\text{T2}} = 1 \Rightarrow ST_{s,R'} \ge d_{s,\text{T1}} + \mathrm{d}_{s,\text{T2}}$$

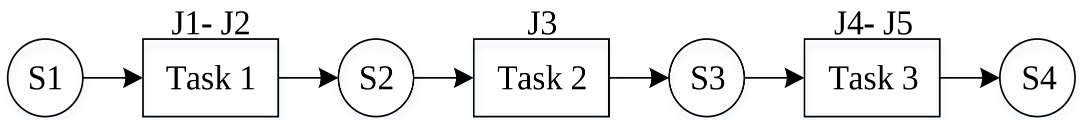
$$Y^{\text{last}}_{R'',\text{T3}} = 1 \Rightarrow ST_{s,R''} \ge d_{s,\text{T1}} + \mathrm{d}_{s,\text{T2}} + \mathrm{d}_{s,\text{T3}}$$

$$Y_{r,t} \wedge \neg Y_{r+1,t} \Rightarrow Y^{\text{last}}_{r,t} \quad \forall r, t$$

$$\bigvee_t \left[\begin{array}{c} Y^{\text{last}}_{r,t} \\ ST_{s,r} \ge \sum_{t'=1}^{t} \mathrm{d}_{s,t'} \end{array}\right], \quad \forall s, r$$

Results (Ex1)

- **Maximum profit over a horizon of 8 h**



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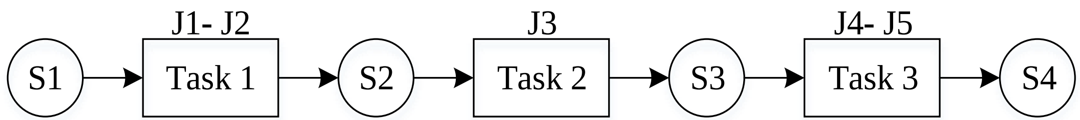
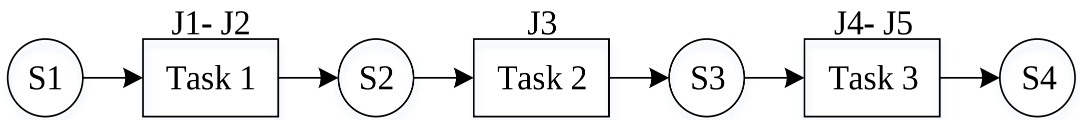


	Binary variables	Total variables	Eqs	CPU (s)	MILP ($)	RMILP ($)
Maximum profit using 5 runs (time horizon length = 8 h)						
Shaik and Floudas (2009)	25	121	222	0.06	1840.1	2982.1
Our	25	87	163	0.02	1840.1	2493.1

- Smaller problem size

Results (Ex1)

- Maximum profit over a horizon of 8 h



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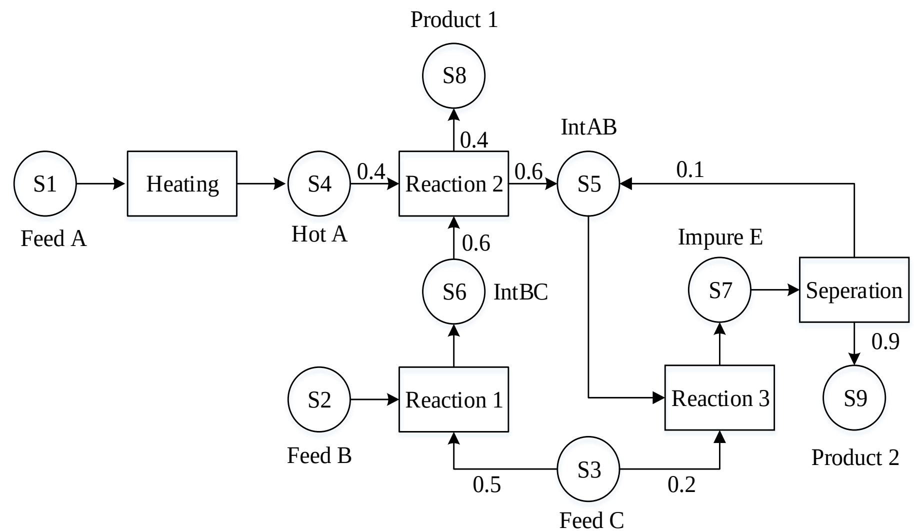
- Smaller problem size
- Tight LP-relaxation

Results (Ex2)

- **Minimum makespan**

Scenario 1: $d_{s8} = d_{s9} = 200$ mu

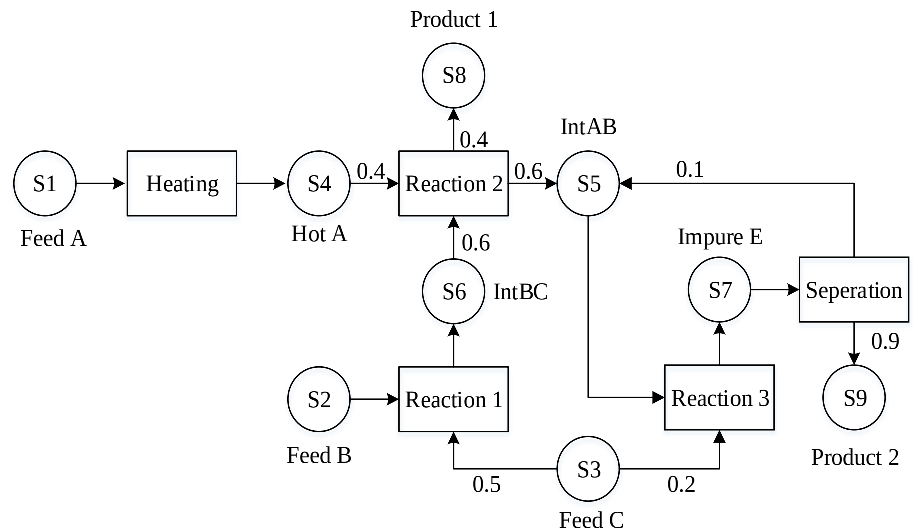
Scenario 2: $d_{s8} = d_{s9} = 300$ mu



Results (Ex2)

- **Minimum makespan**

	Binary variables	Total variables	Eqs	CPU (s)	MILP (h)	RMILP (h)
$d_{S8}=d_{S9}$=200 mu, with 8 events and 8 runs						
SF[a]	64	330	521	0.8	19.7	18.6
Our	64	227	357	0.1	19.7	18.6
$d_{S8}=d_{S9}$=300 mu, with 13 events and 13 runs						
SF[a]	104	535	856	3600[b]	28.7	28.0
Our	104	362	572	743.1	28.7	28.0
[a]Shaik and Floudas (2009), [b]Relative Gap =1.3%,						



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Results (Ex2)

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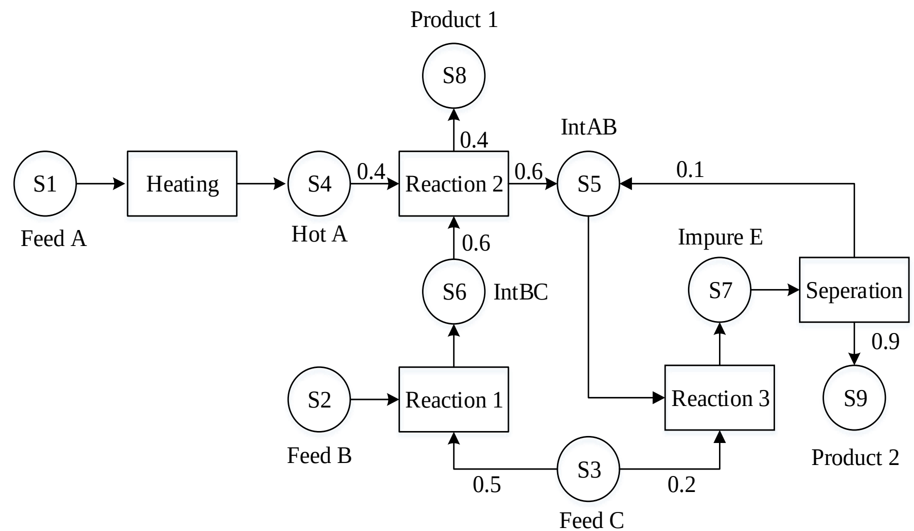
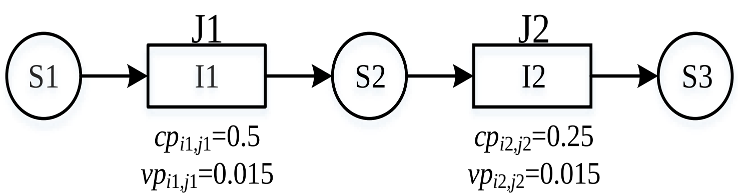


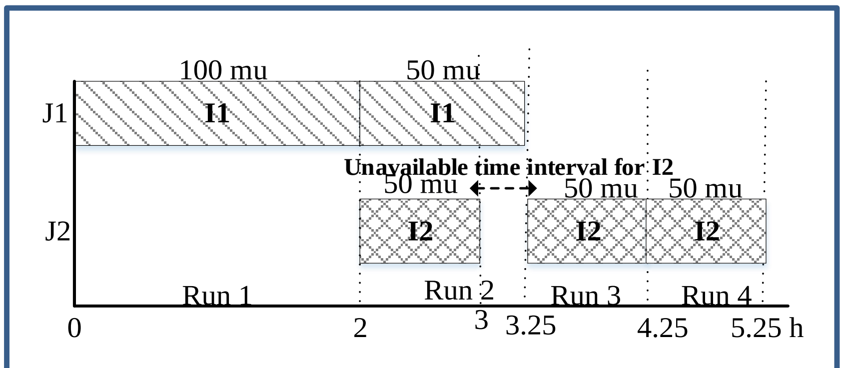
- Smaller problem size
- Lower CPU time

Results (Ex3)

- **The model may result in suboptimal solutions**



The longest element determines the duration of a run: $$LR_{j,r} \leq LR_r \quad \forall j,r$$



Suboptimal

Results (Ex3)

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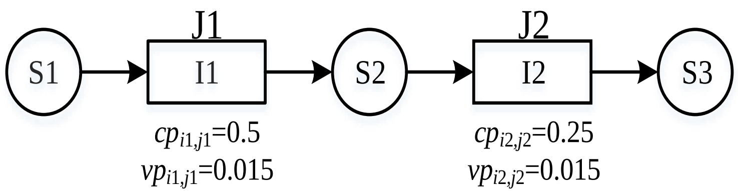

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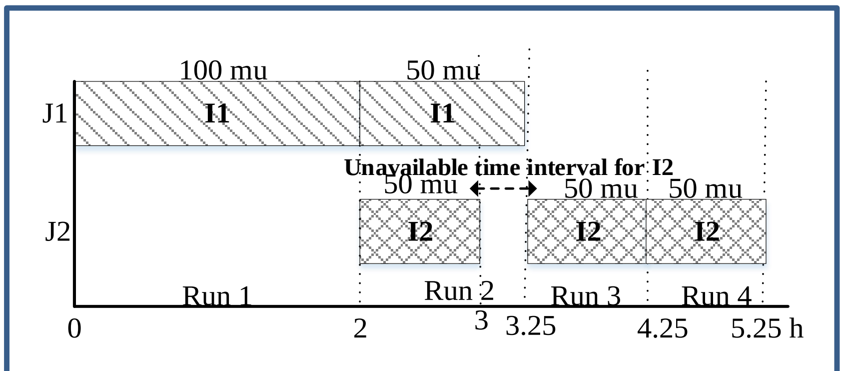
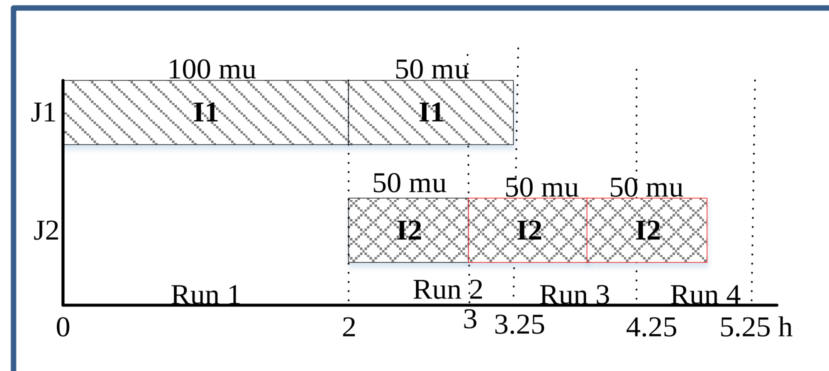


Suboptimal



Optimal

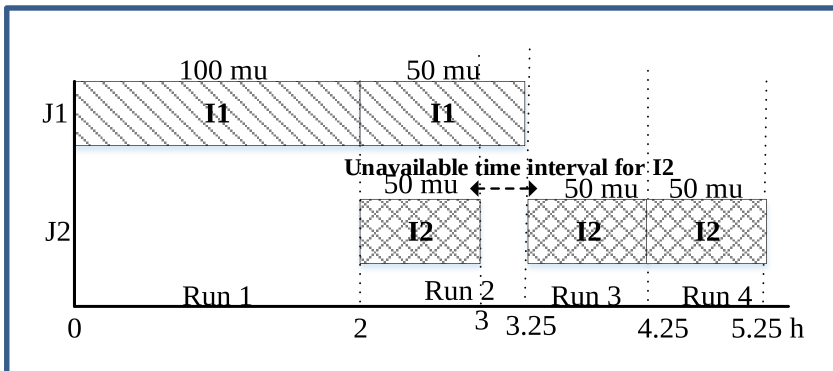
Results (Ex3)

- **The model may result in suboptimal solutions**

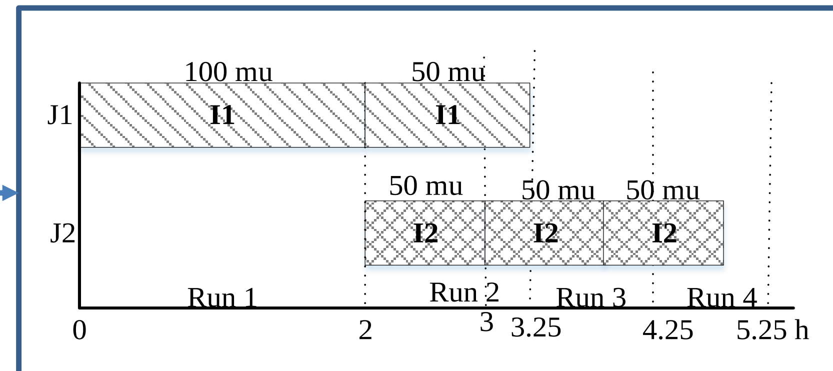
Start time of run r in unit j

$$SR_{j,r} \geq SR_{j,r-1} + LR_{j,r-1} \quad \forall\ j \in J, r \in R|_{r>1}$$

$$SR_{j,r} \geq SR_{j',r-1} + LR_{j',r-1}, \quad \forall\ j \in J_s^c, j' \in J_s^p (j \neq j'), s \in S, r \in R|_{r>1}$$



Suboptimal



Optimal

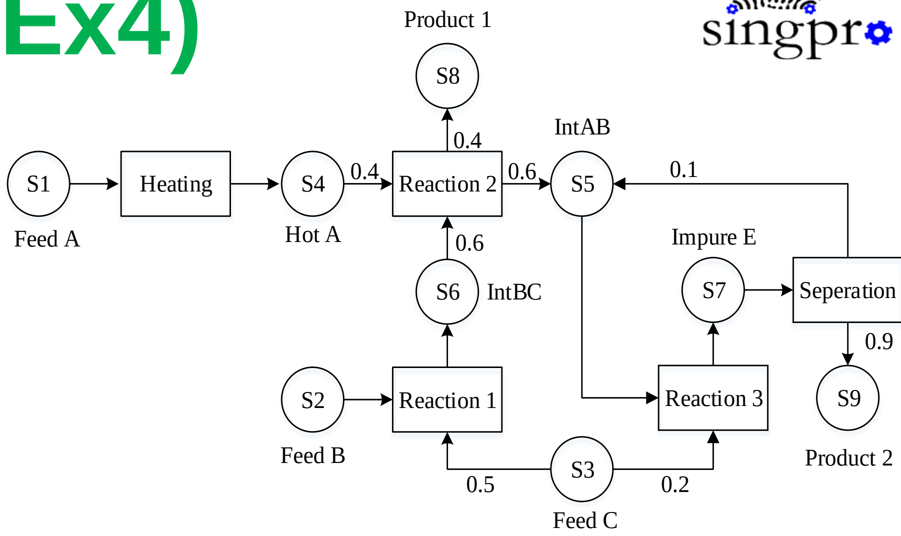
Results (Ex4)

- **Minimum makespan**

Demand should be met during two periods T1 and T2, where $e_{\mathrm{T1}} = 16$ and $e_{\mathrm{T2}} = 26$ h

$d_{\mathrm{T1},S8} = d_{\mathrm{T1},S9} = 100$

$d_{\mathrm{T2},S8} = d_{\mathrm{T2},S9} = 200$



Results (Ex4)

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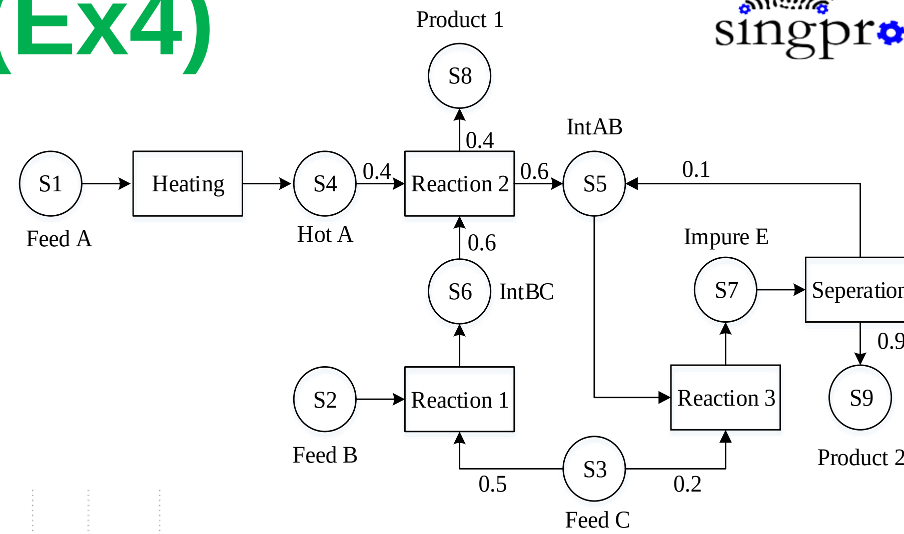
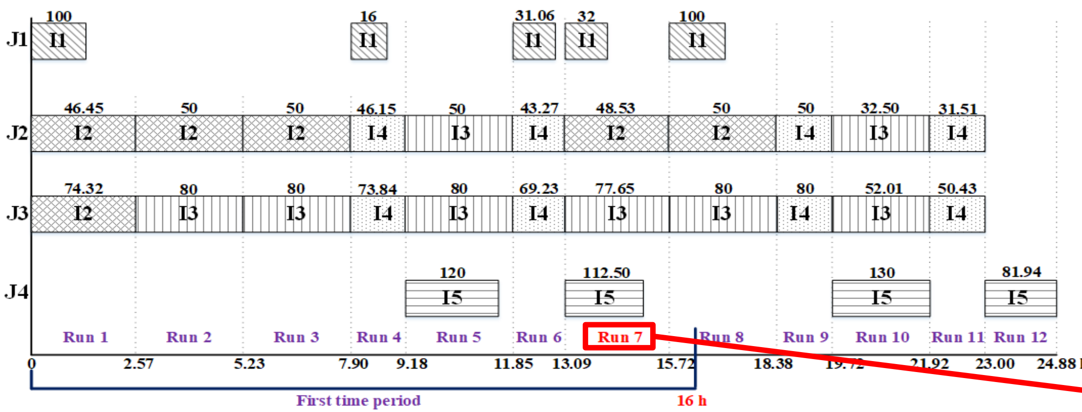
$d_{T2,S8} = d_{T2,S9} = 200$





Last run in the first period

Conclusions

- Novel MILP scheduling formulation for multipurpose batch plants
- The first MILP model fully derived from GDP followed by a convex hull reformulation (no big-M constraints)
- The formulation can be extended to address continuous processes (e.g., grade change optimization)
- Tighter LP-relaxation

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Thank you for your attention!

Any Questions?