Novel Discrete-Time MILP Scheduling Model for Pipeline Systems

Hossein Mostafaei
(hossein.mostafaei@aalto.fi)

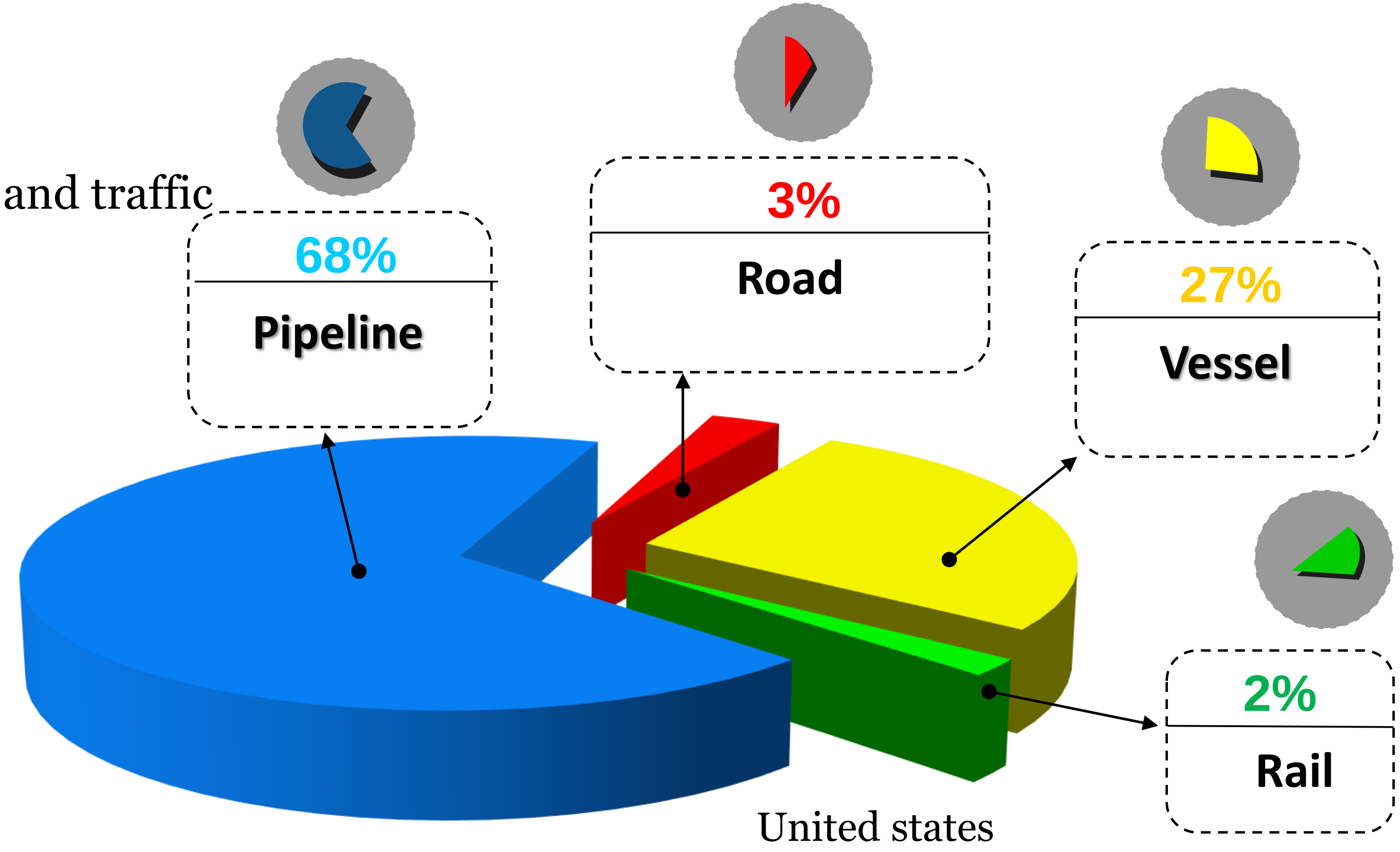
Iiro Harjunkoski
(iiro.harjunkoski@aalto.fi)

Pedro M Castro
(pmcastro@fc.ul.pt)

Problem Importance

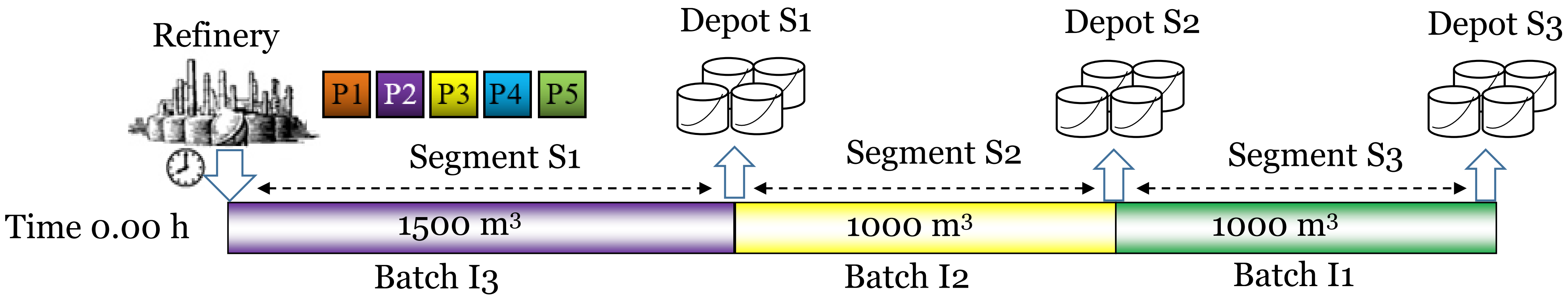
- Advantages of pipelines
  - Multi-product handling
  - Independent from weather and traffic
  - Safe and economic
  - Huge volume of products

- Disadvantages
  - Interface reprocessing cost
  - Low speed



Problem statement

- A pipeline connects a single refinery to multiple depots
- All pipeline segments operates in a single flow direction (from refinery to depots)
- Products inside the pipeline are labeled with batches



- At time T= 0 h, there are three batches I1 (P5), I2 (P3) and I3 (P2) inside the pipeline.
- To be determined
  - Sequence, volume and timing of any injections at refinery
  - Sequence, volume and timing of deliveries to depots
  - Batch sizes

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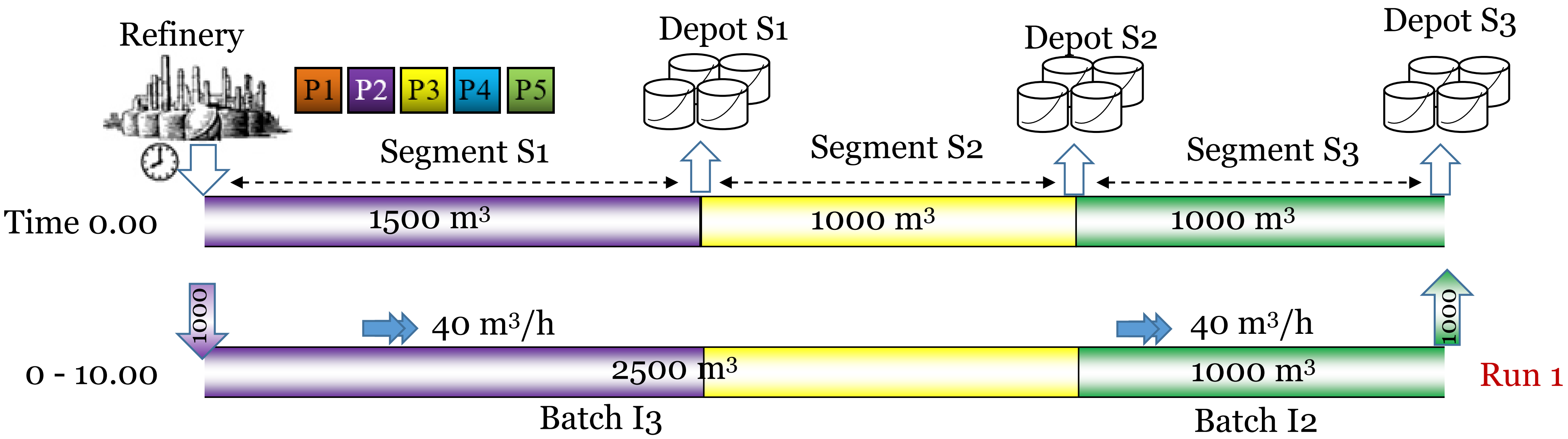

❖ During time interval [0,10] h, product P2 is injected (batch I3) and P5 is delivered to S3.

❖ Batch I2 enters segment S3 and push batch I1 into segment S1

Problem statement

- A pipeline connects a single refinery to multiple depots
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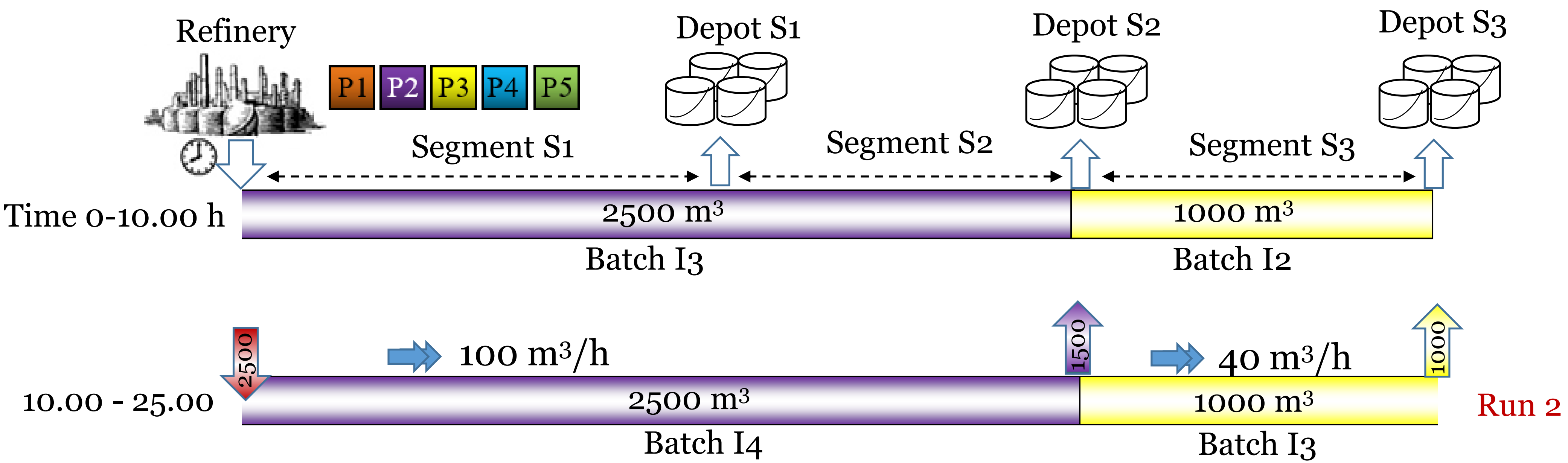
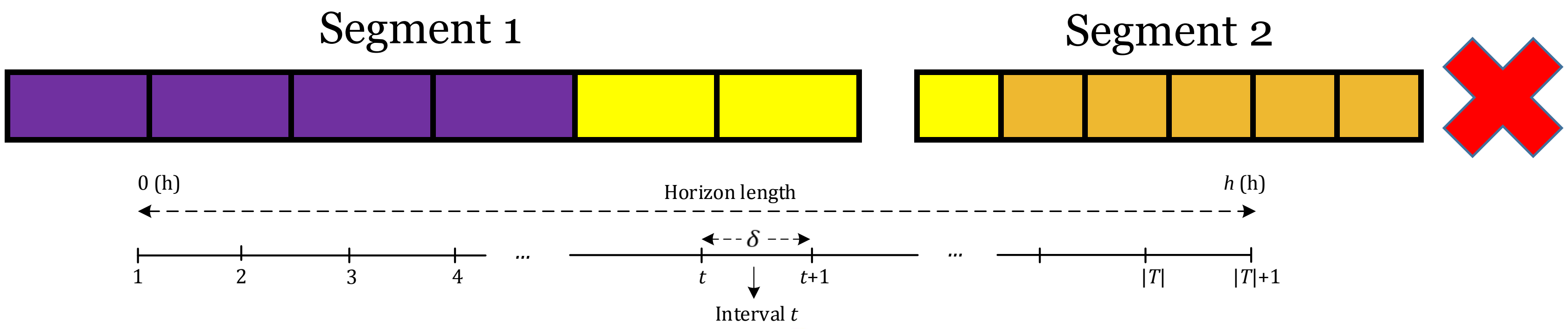


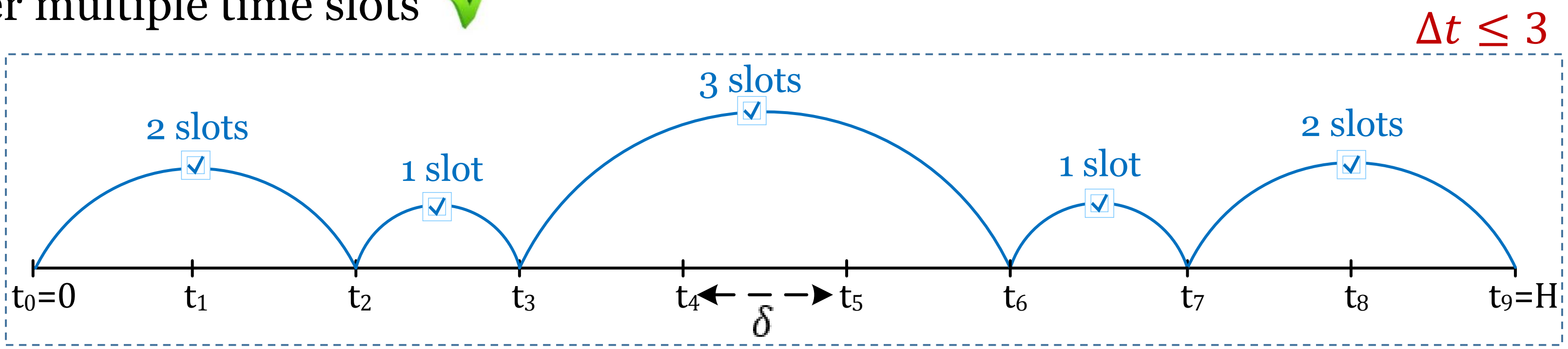
❖ During time interval [10,25] h, new batch I4 (P1) is injected depots S2 and S3 simultaneously receive material

Novelty of the work

- Discrete-time model: Pipeline segments need not to divided into packs of equal sizes



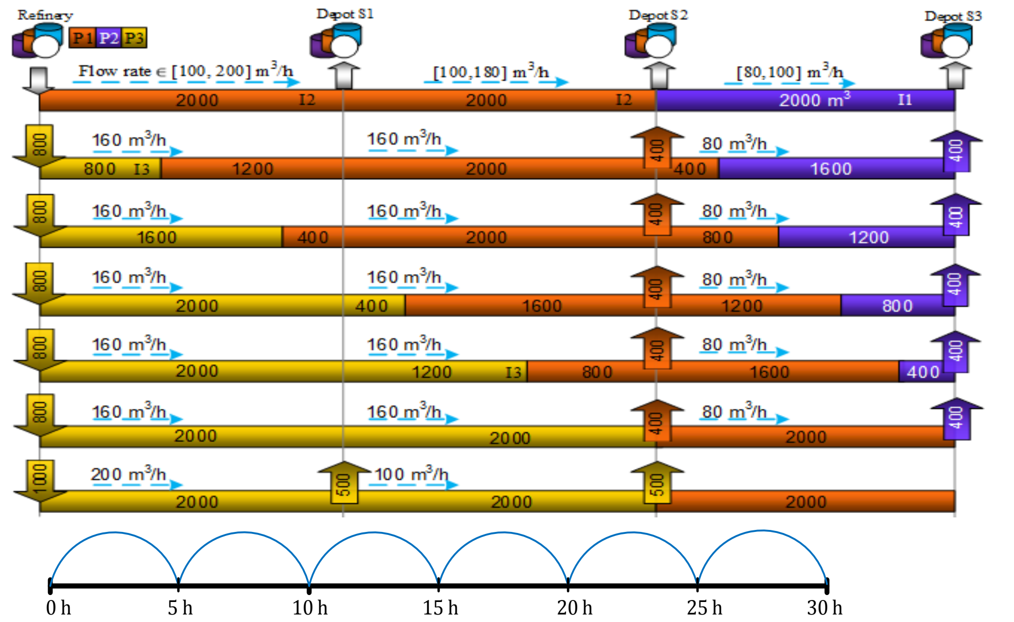
- A pumping run can span over multiple time slots
  - Δt: Spanning limit



Novelty of the work

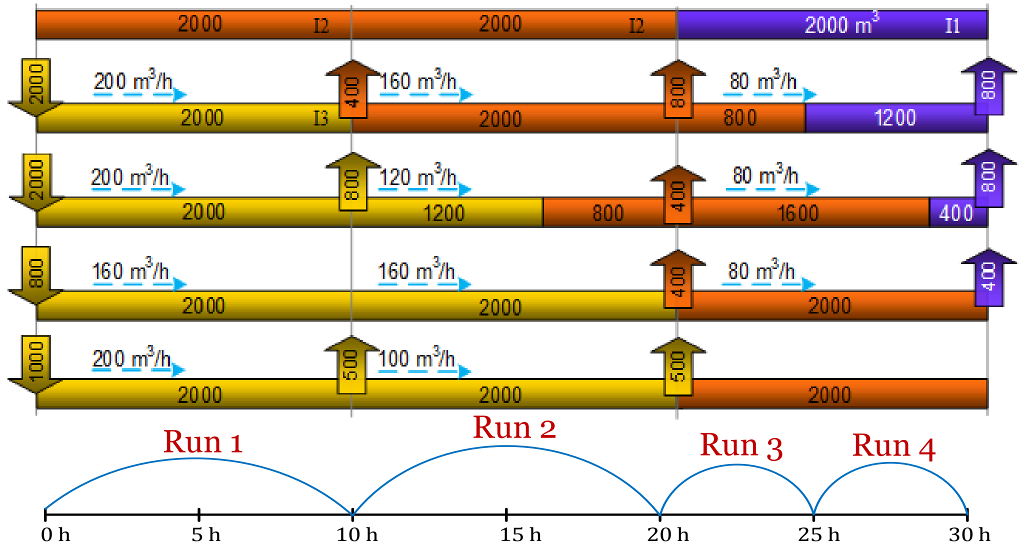
- A pumping run can span over multiple time slots ✓

Spanning limit = 1 slot (5 h), previous approches



Backorder: 400_{p1} and 1000_{p3} in S1

Spanning limit = 2 slots (10 h)



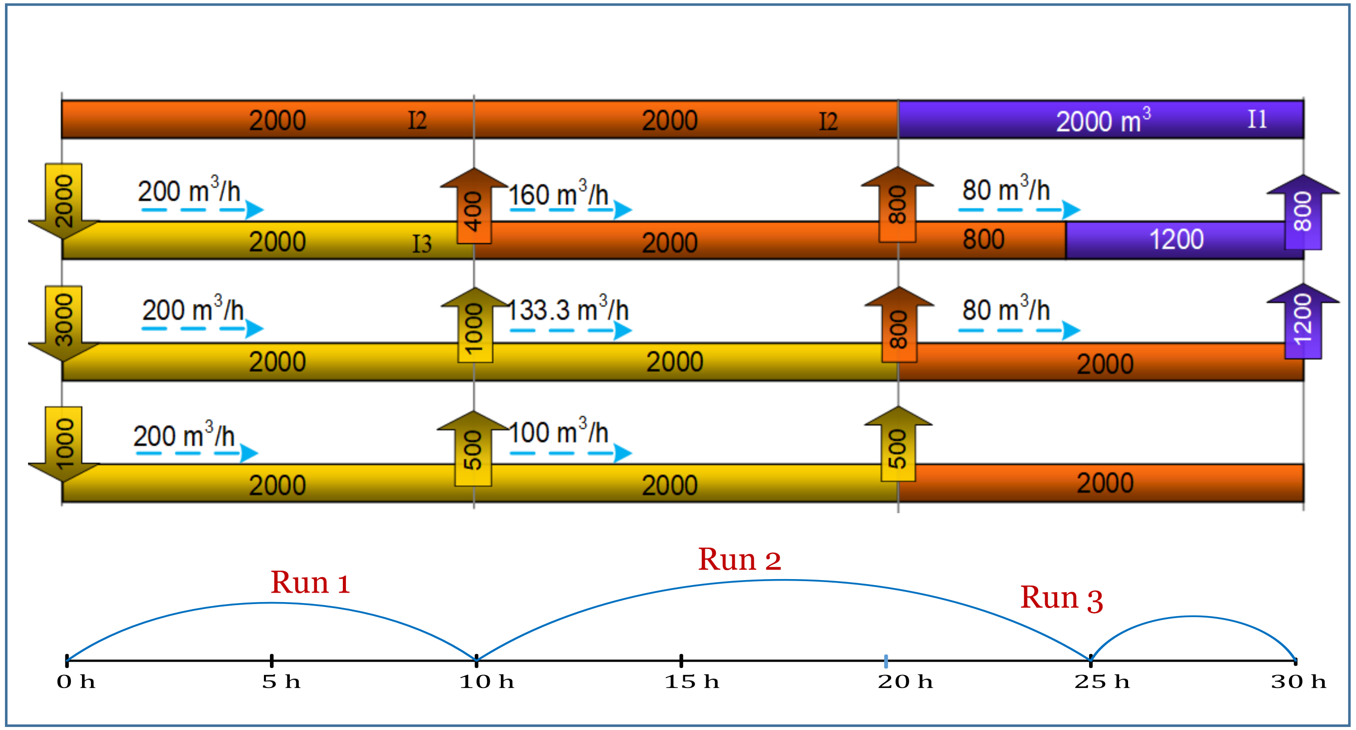
Backorder: 200_{p3} in S1

Novelty of the work

- A pumping run can span over multiple time slots ✓

Spanning limit = 3 slots (15 h)

No backorder

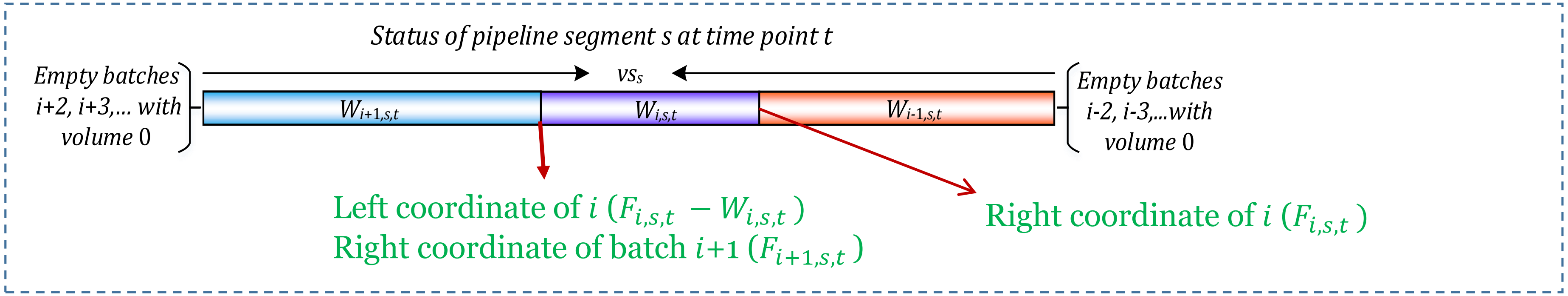


Mathematical formulation

- Location of batches in segments
  - Batch i always follows batch i-1 inside segment s
  - Right coordinate: volume between the origin of segment s and the interface between batches i and $i-1$

$$\boldsymbol{F_{i,s,t}} = \sum_{\substack{i' \in I_s \\ i' \geq i}} \boldsymbol{W_{i',s,t}} \quad \forall \boldsymbol{i} \in \boldsymbol{I_s}, \boldsymbol{s}, \boldsymbol{t}$$

Size of batch i in seg s at t



  - The right coordinate of batch $i+1$ = The left coordinate of batch i

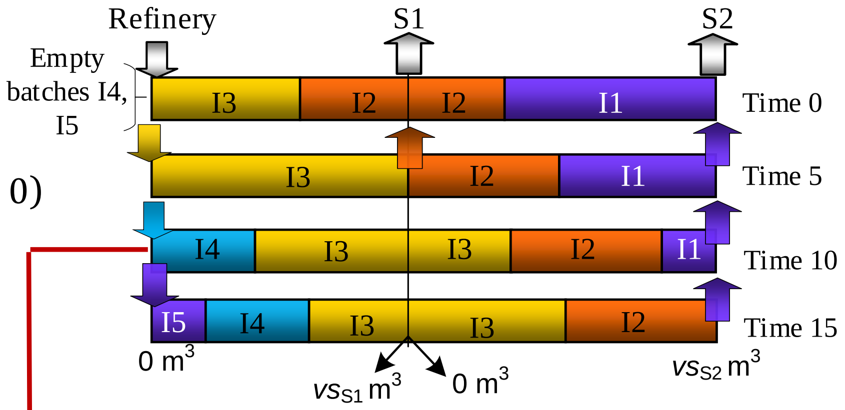
$$\boldsymbol{F_{i,s,t}} - \boldsymbol{W_{i,s,t}} = \boldsymbol{F_{i+1,s,t}} \quad \forall \boldsymbol{i} \in \boldsymbol{I_s}, \boldsymbol{s}, \boldsymbol{t}$$

Mathematical formulation

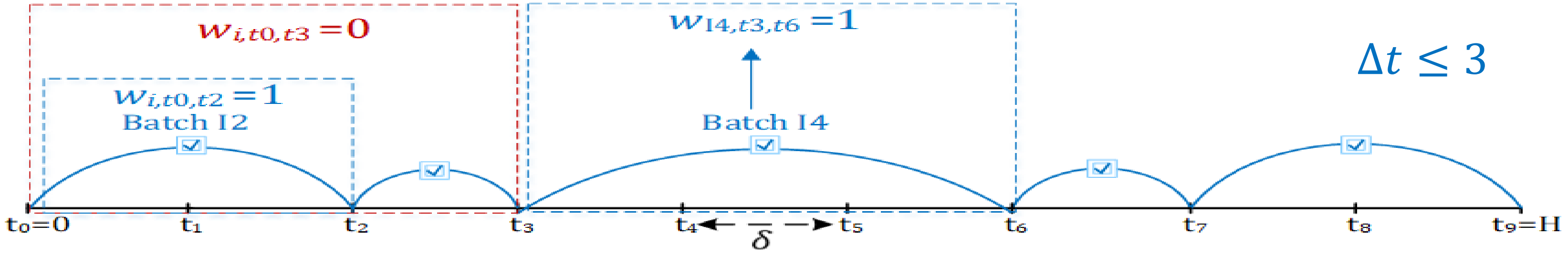
- Injection at refinery
  - In each pumping run only one batch can be injected by the refinery
  - The left coordinate of batch in first segment (s =1) is zero ($F_{i,1,t} - W_{i,1,t} = 0$)
  - $w_{i,t,t'}$ =1 , if batch i is injected during time interval $[t,t']|_{t<t'\le t+\Delta t}$

$$\bigvee_{i\in I^R}\begin{bmatrix} w_{i,t,t'} \\ F_{i,1,t}-W_{i,1,t}=0 \\ v_{min}\le V_{i,t,t'}\le v_{max} \\ L_{t,t'}=\delta\times(t'-t)\end{bmatrix} \underline{\bigvee} \begin{bmatrix} w^{\text{no } i}_{t,t'} \\ F_{i,1,t}-W_{i,1,t}\le vs_1 \\ V_{i,t,t'}=0 \quad \forall i\in I^R \\ L_{t,t'}=0\end{bmatrix} \qquad t<t'\le t+\Delta t$$

Duration of a run at refinery during time interval $[t,t']|_{t<t'\le t+\Delta t}$



At time 10 h, only batches I4 or I5 have left coordinate 0



$\Delta t \le 3$

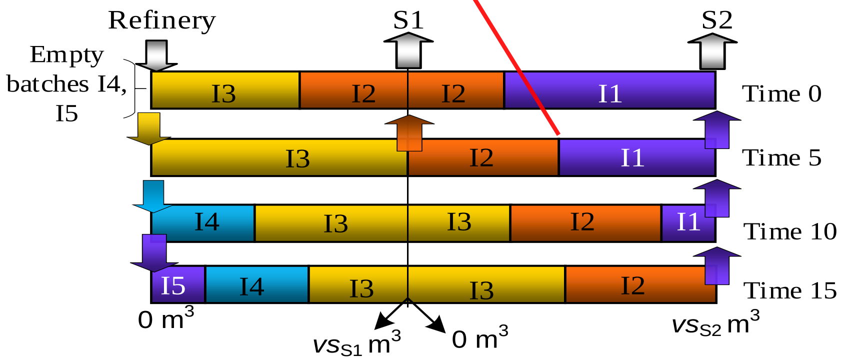
Mathematical formulation

- Injection at refinery

- Delivery to depots
  - In each pumping run only one batch can be delivered to depot s
  - The right coordinate of batch in segment s reaches end of that segment ($F_{i,s,t} = vs_s$)
  - $x_{i,s,t,t'}$ =1 , if batch i is delivered to depots s during time interval [t,t']

At time 0, only batches I1 has right coordinate vs_{S2}

$$\bigvee_{i\in I_s}\begin{bmatrix} x_{i,s,t,t'} \\ F_{i,s,t} \ge vs_s \\ d_{s,min} \le D_{i,s,t,t'} \le d_{s,max}\end{bmatrix} \underline{\bigvee} \begin{bmatrix} x^{no\ i}_{s,t,t'} \\ F_{i,s,t,t'} \ge 0 \ \ \forall i \in I_s \\ D_{i,s,t,t'} = 0 \ \ \forall i\in I_s\end{bmatrix} \quad \forall s \in S, t < t' \le t+\Delta t$$

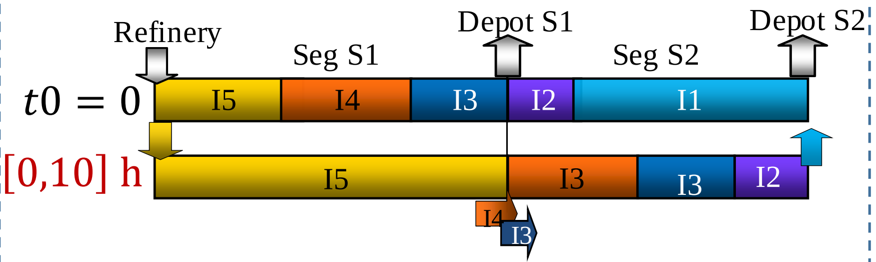


Mathematical formulation

- Feeding segment
  - Multiple batches can enter a segment
  - Segment s feeds segment s-1

$v_{s,t,t'}$ = 1, segment s is active
$u_{i,s,t,t'}$ = 1, segment s is fed by batch i
$S_{i,s,t,t'}$ = volume of batch i entered segment s
$L_{s,t,t'}$ = activity length of segment s

$$\underline{\bigvee} \left[\begin{array}{c} \boldsymbol{v_{s,t,t'}} \\ \Delta \le \boldsymbol{L_{s,t,t'}} \le \boldsymbol{h} \\ \boldsymbol{L_{t,t'} - L_{s,t,t'} = 0} \\ \bigvee_{i\in I_{s-1}} \left[\begin{array}{c} \boldsymbol{u_{i,s,t,t'}} \\ \boldsymbol{s_{s,min} \le S_{i,s,t,t'} \le s_{s,max}} \\ \boldsymbol{F_{i,s-1,t'} \ge vs_s} \end{array}\right] \end{array}\right] \underline{\bigvee} \left[\begin{array}{c} \boldsymbol{\neg v_{s,t,t'}} \\ \boldsymbol{L_{s,t,t'} = 0} \\ \bigvee_{i\in I_{s-1}} \left[\begin{array}{c} \boldsymbol{\neg u_{i,s,t,t'}} \\ \boldsymbol{S_{i,s,t,t'} = 0} \\ \boldsymbol{F_{i,s-1,t'} \ge 0}\end{array}\right]\end{array}\right] \quad \forall \boldsymbol{s}|_{s\ge 2}, \forall \boldsymbol{t,t'}|_{t<t'\le t+\Delta t}$$



$$L_{S1,t0,t1} = L_{S2,t0,t1} = L_{t0,t1} = 10\text{ h}$$

$$u_{I3,s2,t0,t1} = u_{I4,s2,t0,t1} = 1$$

- Inventory and meeting demand (multiple due dates)

$$IS_{p,s,t} = IS_{p,s,t-1} + \sum_{\substack{t'\in T \\ t'<t\le t'+\Delta t}} \sum_{i\in I_s} DP_{i,p,s,t',t} - LM_{p,s,t} \quad \forall p,s,t$$

inventory of p in depot s at time point t

$$\sum_{\substack{t\in T \\ ap_{tp}\ge a_t}} LM_{p,s,t} + BR_{p,s,tp} \ge \sum_{tp'\in TP}^{tp} Demd_{p,s,tp'} \quad \forall p,s,tp$$

Volume to markets from S

Demand at period tp

Mathematical formulation

- Forbidden sequences
  - At injection point
    - Batch *i* is always injected immediately after batch *i-1* at the refinery

$$FS_{p,p'} \Rightarrow \neg(Y_{i,p} \wedge Y_{i-1,p'}) \quad \forall i \in I_s^{\text{new}}, p, p' \in P$$

$$y_{i,p} + y_{i-1,p'} \leq 2 - FS_{p,p'} \quad \forall i \in I_s^{\text{new}}, p, p' \in P$$

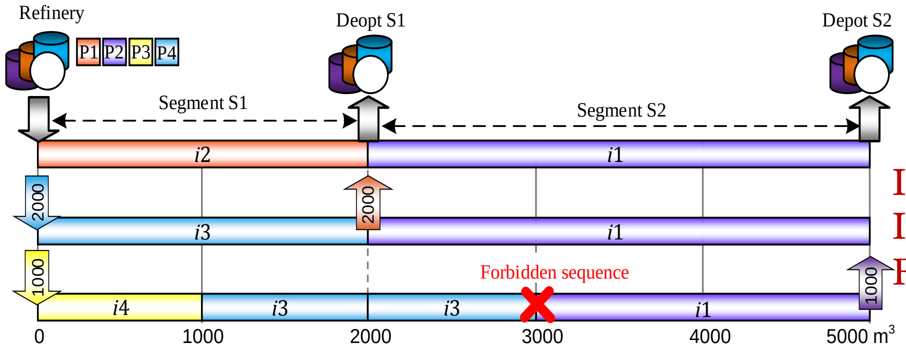
1 if batch i conveys product p

  - At junction of segments
    - Empty batches can be generated by completely transferring the content of batches to middle depots
    - I2 becomes empty at first run and I3 and I1 touch

$$r_{i',s} \wedge y_{i,p} \wedge y_{i',p'} \wedge \neg\left(\bigvee_{j \geq i'+1}^{i-1} r_{j,s}\right) \wedge FS_{p,p'} \Longrightarrow \neg r_{i,s} \quad \forall i, i', p, p', s \geq 2$$

$$r_{i,s} + r_{i',s} \leq \sum_{j=i'+1}^{i-1} r_{j,s} - y_{i,p} - y_{i',p'} - FS_{p,p'} + 4 \qquad \forall i, i', p, p', s \geq 2$$

1 if batch i is in segment s



I1→ P2
I3 →P4
Forbidden

P4-P2 and P2-P4 are forbidden sequences ($FS_{P2,P4}=FS_{P4,P2}=1$)

Empty batch I2 moves at the interface of I3 and I1

Mathematical formulation

- Filler batch
  - Pipeline segments are always full of products, so there is always a batch with positive volume inside

$$\bigvee_{i\in I_s}\left[\begin{array}{c} z_{i,s,t}\\ d_{s,\min}\le W_{i,s,t}\le vs_s\end{array}\right]\quad \forall s\in S,t\in T$$

MILP

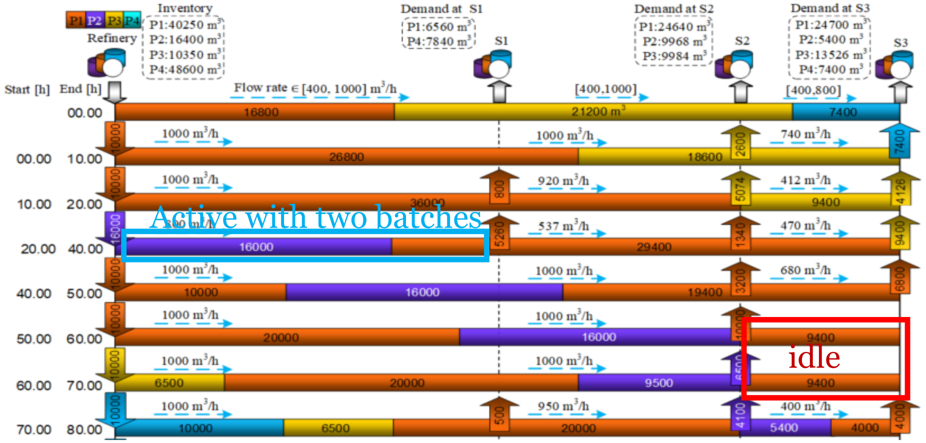
$$\sum_{i\in I_s} z_{i,s,t}\ge 1 \quad \forall s\in S,t\in T$$

$$d_{s,\min} z_{i,s,t}\le W_{i,s,t}\le vs_s z_{i,s,t}\quad \forall i\in I_s, s\in S,t\in T$$

  - Contamination at the interface ↑ while the segment stays idle
    - One batch when segment is idle

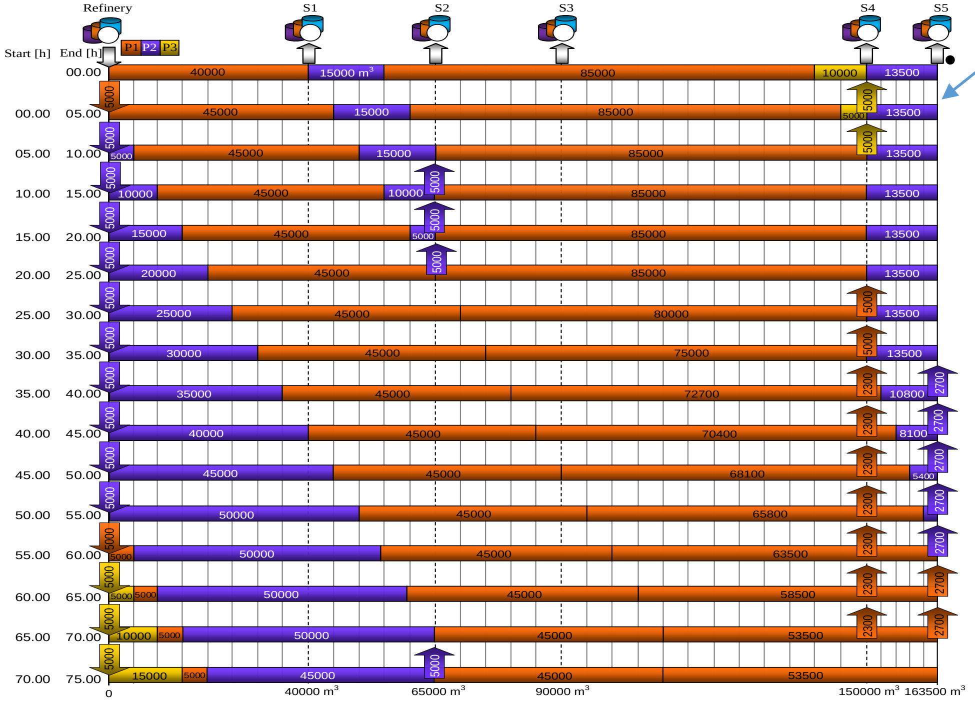
$$\neg \bigvee_{\substack{t'\in T\\ t'<t\le t'+\Delta t}} v_{s,t',t} \wedge \bigvee_{i''\in I^{\mathrm{R}}}\ \bigvee_{\substack{t'\in T\\ t'<t\le t'+\Delta t}} w_{i'',t',t} \Rightarrow \neg\left(z_{i,s,r}\wedge z_{i',s,r}\right)\quad \forall i,i'\in I_s, s\in S,t\in T$$

$$z_{i,s,t}+z_{i',s,t}\le 2-\sum_{i''\in I^{\mathrm{R}}}\ \sum_{\substack{t'\in T\\ t'<t\le t'+\Delta t}} w_{i'',t',t}+\sum_{\substack{t'\in T\\ t'<t\le t'+\Delta t}} v_{s,t',t}\quad \forall i,i'\in I_s, s\in S,t\in T$$



Results

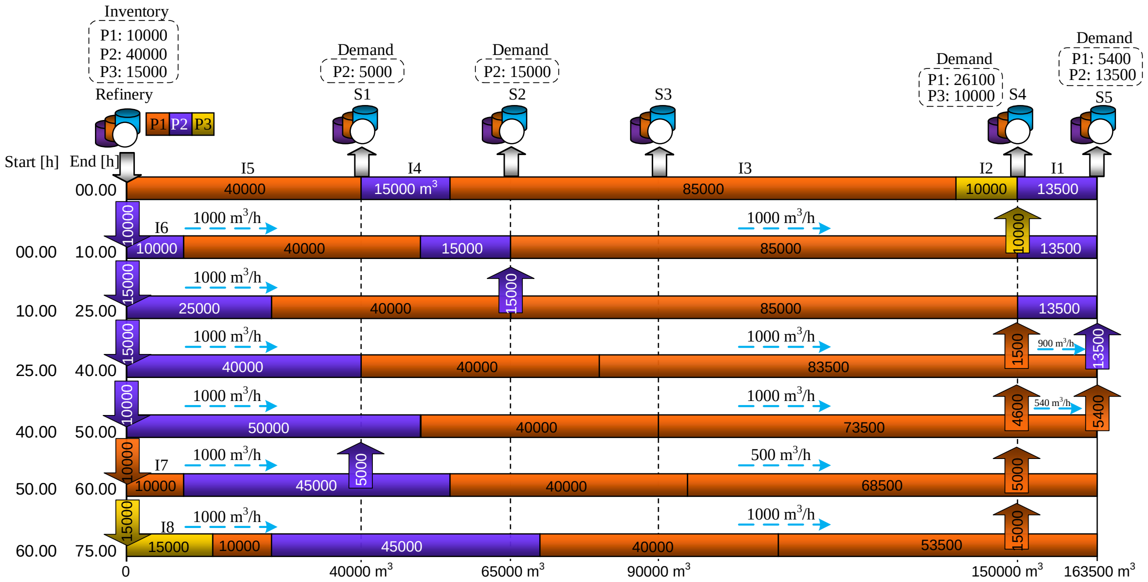
- Comparison to a discrete-model of Rejowski and Pinto (2004), Pump operating cost
  - Rigorously handles forbidden product sequences and filler products



Rejowski and Pinto (2004)

Rejowski and Pinto (2004)

Segments S1-S4 into packs of 5000 m3, S5 into 2700 m3

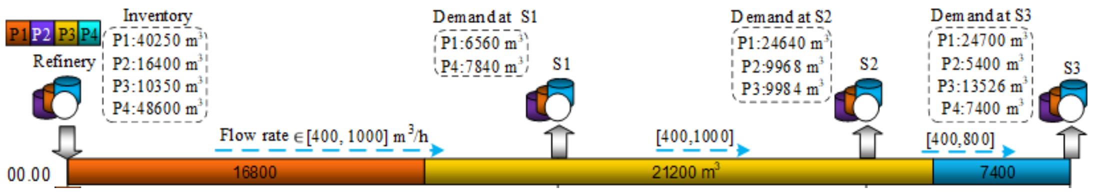


Pump operating cost: $6000 vs. $8000

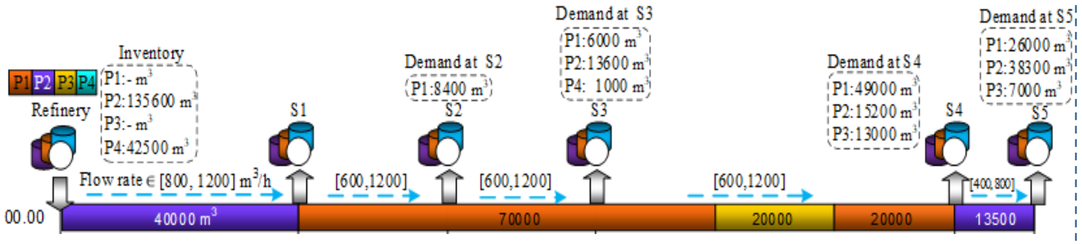
CPU: 1015 s vs. 4780 s

Results

- Comparison to the continuous-time model of Castro and Mostafaei (2019), CM
  - Similary to the proposed model rigorously handles forbidden product sequences and filler products
  - Objective: Flow restart cost



	$\|T\|-1$	CPU (s)	Binary variables	Total variables	Eqs	MILP ($)	RMILP ($)
CM	8	148.1	968	2757	4480	106420	1520.7
	9	325.5	1087	3088	5020	5480	1351.7
	10	2126.2	1206	3419	5560	5480	1216.5
Our	12 ($\Delta t = 1$)	18.20	737	2912	5649	5480	2395.1
	12 ($\Delta t = 2$)	506.4	1166	4870	8014	5480	2210.0



	$\|T\|-1$	CPU (s)	Binary variables	Total variables	Eqs	MILP ($)	RMILP ($)
CM	7	70	1030	2635	3962	416350	7037.88
	8	155	1176	2994	4508	16350	6158.14
	9	368	1322	3553	5045	16350	5473.90
Our	12 ($\Delta t = 1$)	20.6	904	3541	6241	416350	8978.3
	12 ($\Delta t = 2$)	278	1248	4801	8041	16350	7491.52

- ➢ Same optimal cost, Tighter linear relaxation, Fast
- ➢ Same optimal cost, Tighter linear relaxation, Slightly slower

Conclusions

- **Novelty**
  - No need to divide pipeline segments into packs of equal sizes
  - Pumping runs can span over multiple time slots
  - Not need to postulate the number of pumping runs
  - Easily handles due date constraints (no need for extra binary variables)
- **Operational rules**
  - Handels filler batch constraints
  - Handels forbidden product sequences
- **Results**
  - Fewer number of pumping runs compared to other discrete-time models
  - Tighter LP relaxation
  - Applicable to real-life problems