Industrial-scale selective maintenance optimization using bathtub-shaped failure rates

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Content

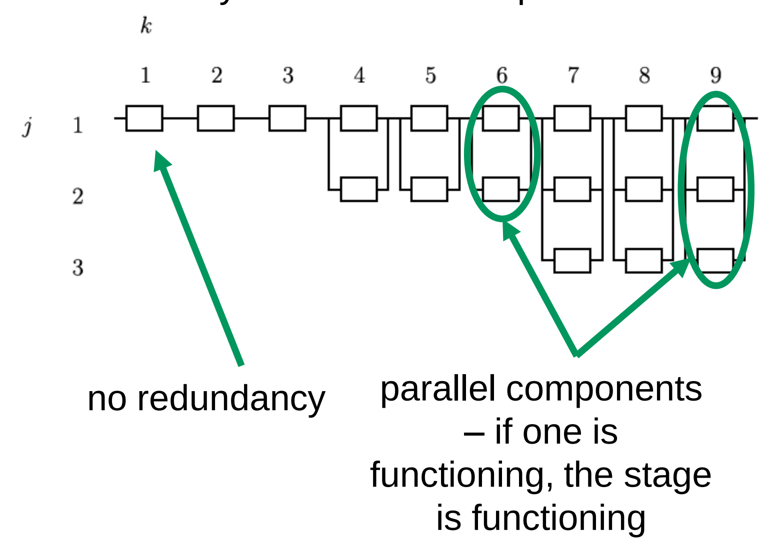
- **Introduction**
  - Problem description
  - Motivation
- **Bathtub-shaped failure models**
- **Selective maintenance optimization**
  - Replacement models
  - Replacement-repair models
- **Conclusions**

Optimization problem

- **A plant has a large number of replaceable / repairable components**
  - Industrial reference: a biorefinery has 600 frequency converters
- **Which to be maintained during a periodic maintenance shutdown?**
  - Industrial reference: these are organized once in 18-24 months
- **Two conflicting objectives**
  - maximize the reliability of the plant for the next operation window
  - minimize the maintenance costs

→ Selective maintenance optimization

Reliability modeling of a system with 18 components



The problem appears in…

Chemical plants

Power plants

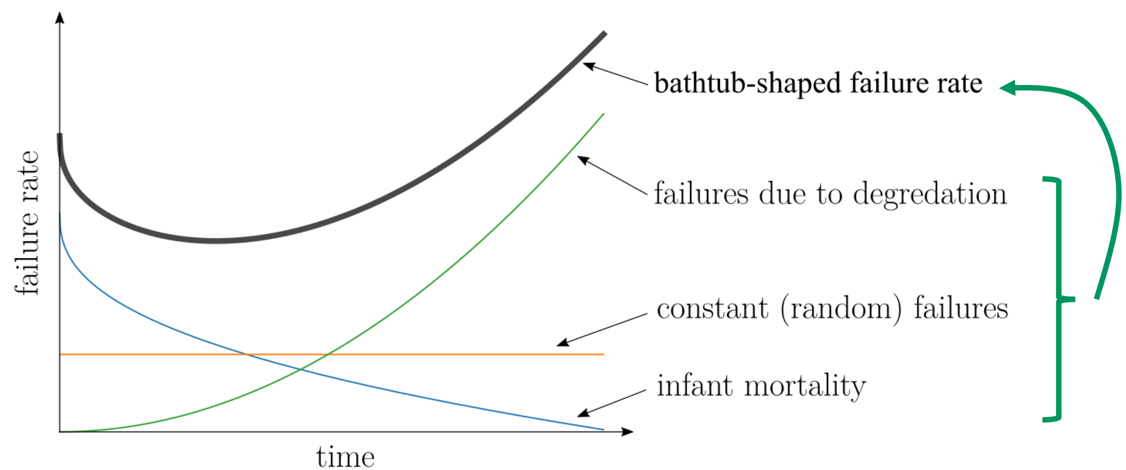
Assembly lines

Transport systems

Graphs: pixabay.com

Motivation 1

- **The selective maintenance optimization literature lacks of data-based studies (Cao et al., 2018)**
  - Typically, the starting point has been (arbitrarily chosen) Weibull parameters
  - Bathtub-shaped failure rates are not considered



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Motivation 2

- **Largest problems solved to optimality in the literature**
  - 200 components with one maintenance action (Galante & Passannanti, 2009)
  - Around 25 components with two or more maintenance actions (Lust et al., 2009, Diallo et al., 2018)
- **The size of the industrial reference**
  - The biorefinery plant has around 600 components

→ A gap between studied problems sizes in the literature and industrial problems

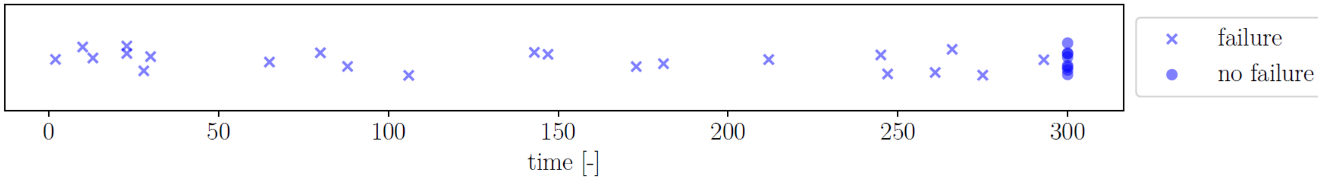
Our contributions

- **Data-driven approach using two bathtub-shaped failure models (Jiang, 2013, Sarhan & Apaloo, 2013)**
- **Reducing the computational cost**
  - Convexification of optimization model
  - Data-based preassignment of variables corresponding to components, the replacement of which is not sensible

Bathtub-shaped failure models

Fitting failure models to data

Experimental data of failure times (Meeker and Escobar, 1998)



- **The failure model by Jiang (2013)**
  - Failure rate $h(t) = \frac{\beta}{t+\eta} + \frac{1}{\gamma - t}$
  - Cumulative failure distribution $\mathrm{F}(t) = 1 + \frac{1 - t/\gamma}{(1+t/\eta)^{\beta}}$
  - Parameters β, γ, η
- **Maximization of the log likelihood, $\log \mathcal{L}$, of the failure model**

$$\max_{\beta,\gamma,\eta} \quad \log \mathcal{L} = \sum_{i=1}^{n} [d_i \log h(t_i) + \log R(t_i)],$$

where
n is the number of data points
t_i is the age of the i^{th} component
d_i indicates whether the i^{th} component failed

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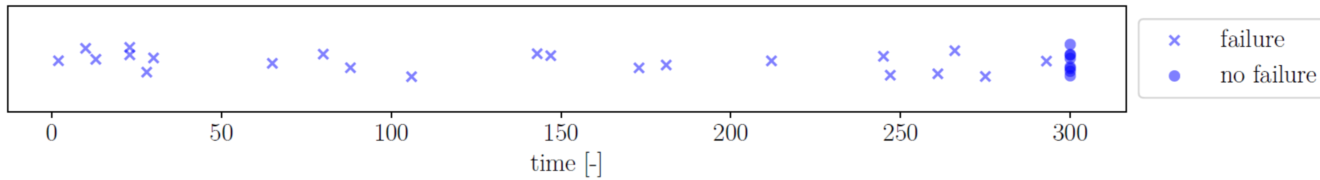
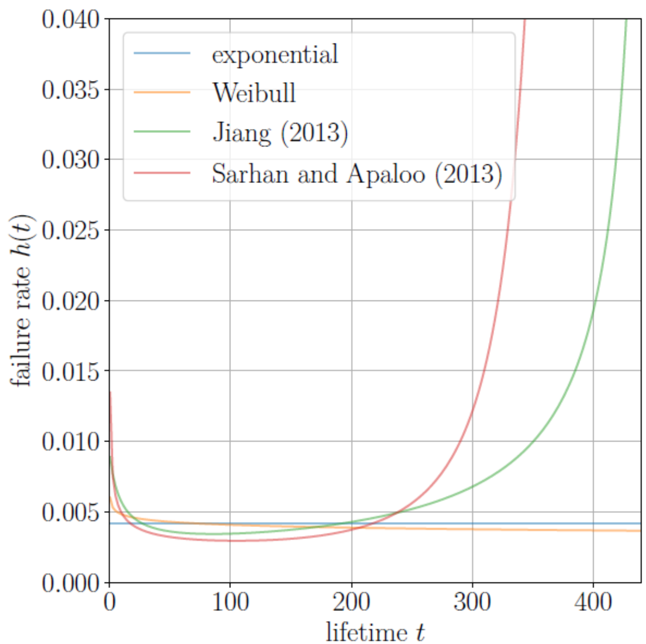
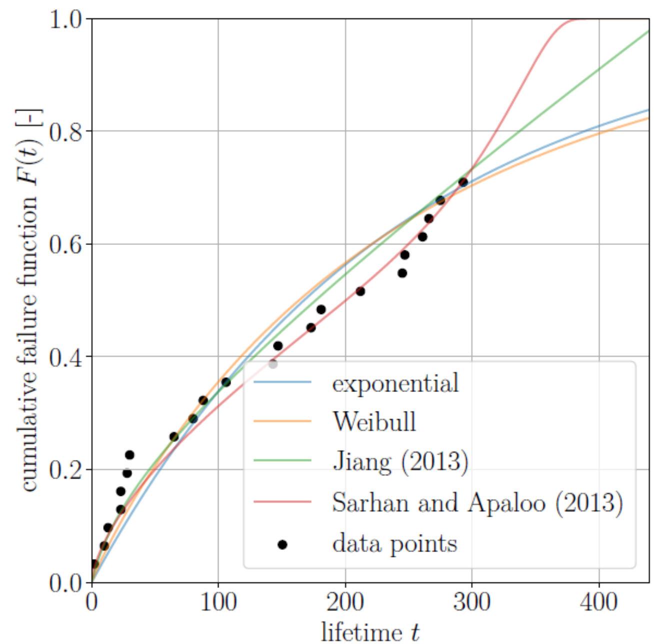
Fitting failure models to data

Experimental data
of failure times
(Meeker and Escobar, 1998)







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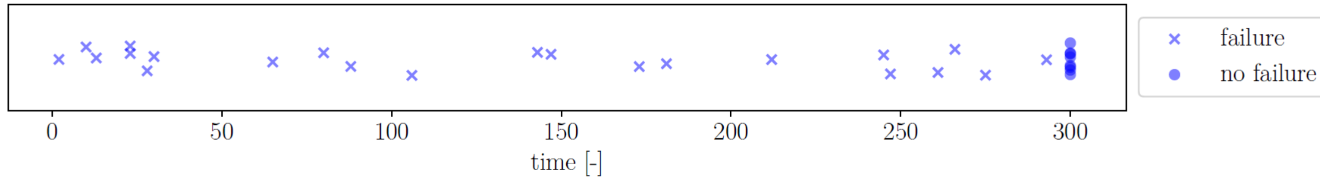
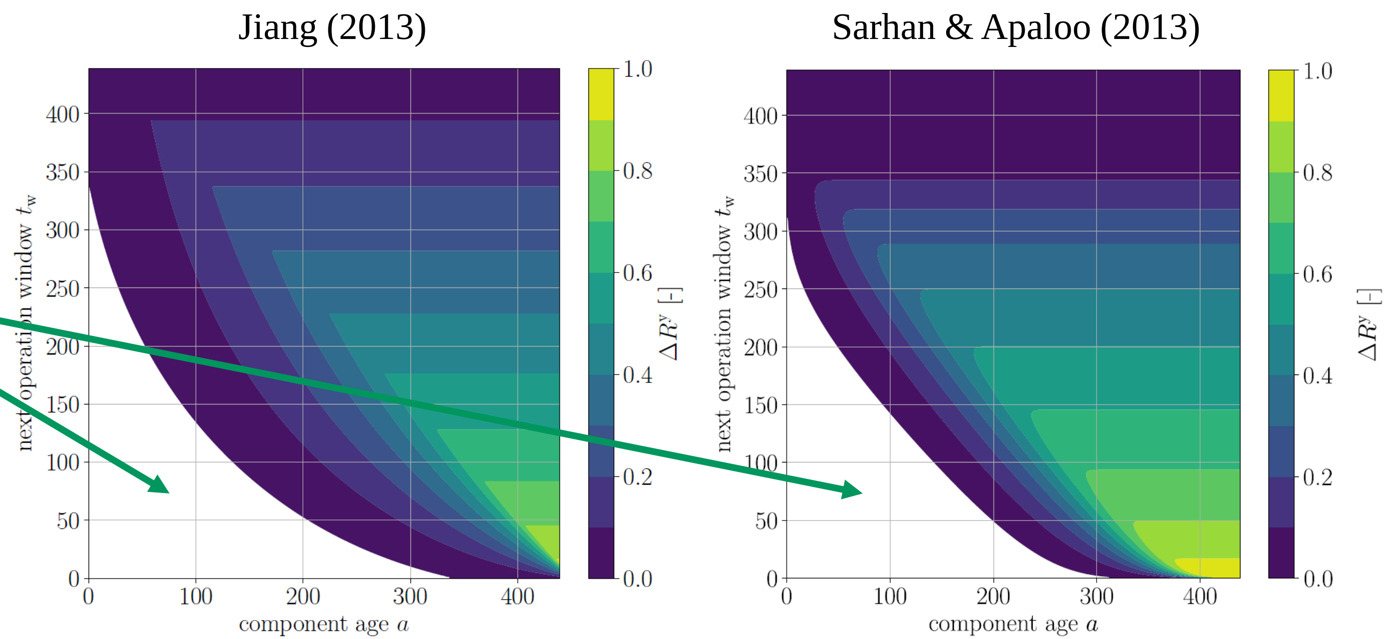
Change in reliability if replaced, ΔR^y

Experimental data of failure times (Meeker and Escobar, 1998)

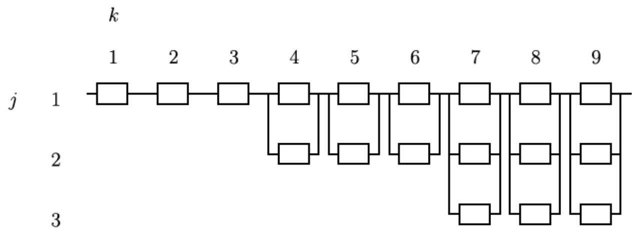


The change in reliability is negative, due to the infant mortality period
→ replacement is not sensible



Selective maintenance optimization

Non-convex replacement model



Sets

K	Set of stages
J_k	Set of parallel components in stage k

Parameters

$R^0_{k,j}$	Reliability of component (k,j) if not replaced
$R^y_{k,j}$	Reliability of component (k,j) if replaced
$\Delta R^y_{k,j}$	$R^y_{k,j} - R^0_{k,j}$ (improvement in reliability)

Variables

R'_k	Reliability of stage k
$y_{k,j}$	Binary variable defining if component (k,j) is replaced
p	Number of maintenance personnel involved in the maintenance operations

$$\max_{\mathbf{y},p} \quad R_{\text{sys}}, -c_{\text{tot}}$$

subject to

System reliability constraint

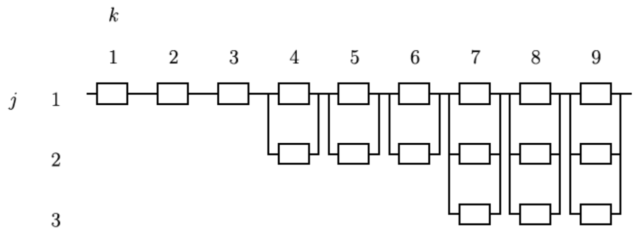
Stage reliability constraint

Total cost constraint

Total duration constraint

Personnel constraint

Non-convex replacement model



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$$\max_{\mathbf{y},p} \quad R_{\text{sys}}, -c_{\text{tot}}$$

subject to

$$R_{\text{sys}} = \prod_{k \in K} R'_k$$

Stage reliability constraint

Total cost constraint

Total duration constraint

Personnel constraint

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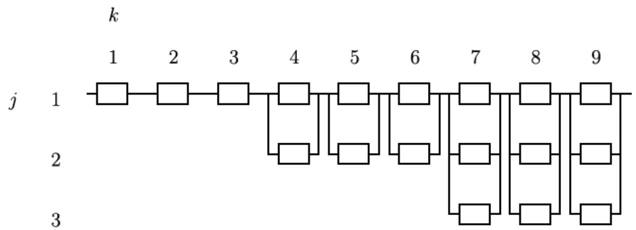
Non-convex replacement model



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Variables

R'_k	Reliability of stage k
$y_{k,j}$	Binary variable defining if component (k,j) is replaced
p	Number of maintenance personnel involved in the maintenance operations

$$\max_{\mathbf{y},p} \quad R_{\text{sys}}, -c_{\text{tot}}$$

subject to

$$R_{\text{sys}} = \prod_{k \in K} R'_k$$

$$R'_k = 1 - \prod_{j \in J_k} \left(1 - R^0_{k,j}(1 - y_{k,j}) - R^y_{k,j} y_{k,j}\right)$$

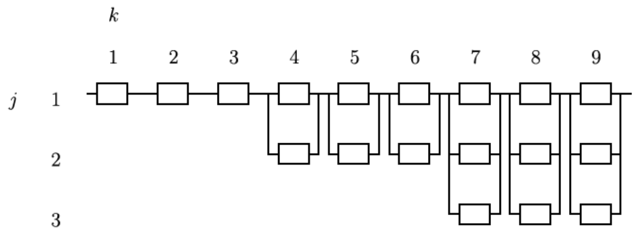
$$= 1 - \prod_{j \in J_k} \left(1 - R^0_{k,j} - \Delta R^y_{k,j} y_{k,j}\right), \quad k \in K$$

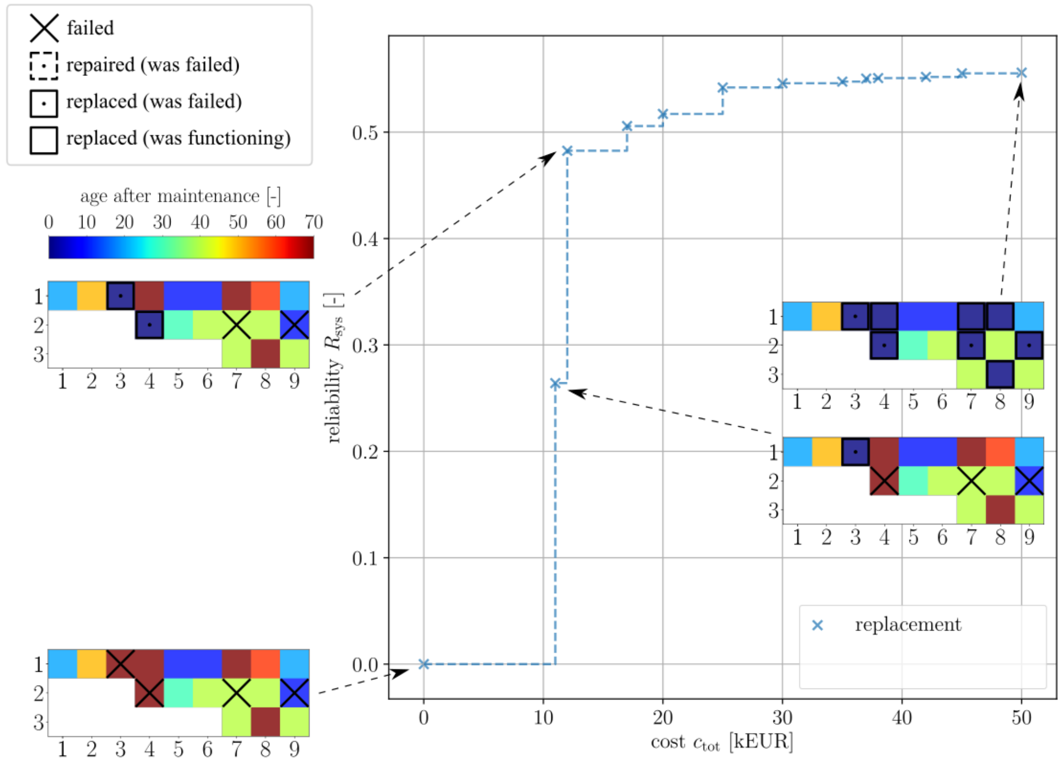
Total cost constraint

Total duration constraint

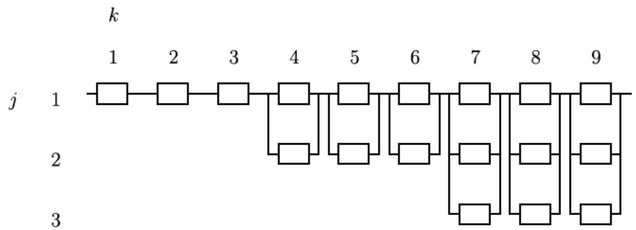
Personnel constraint

Illustrative example





Illustrative example



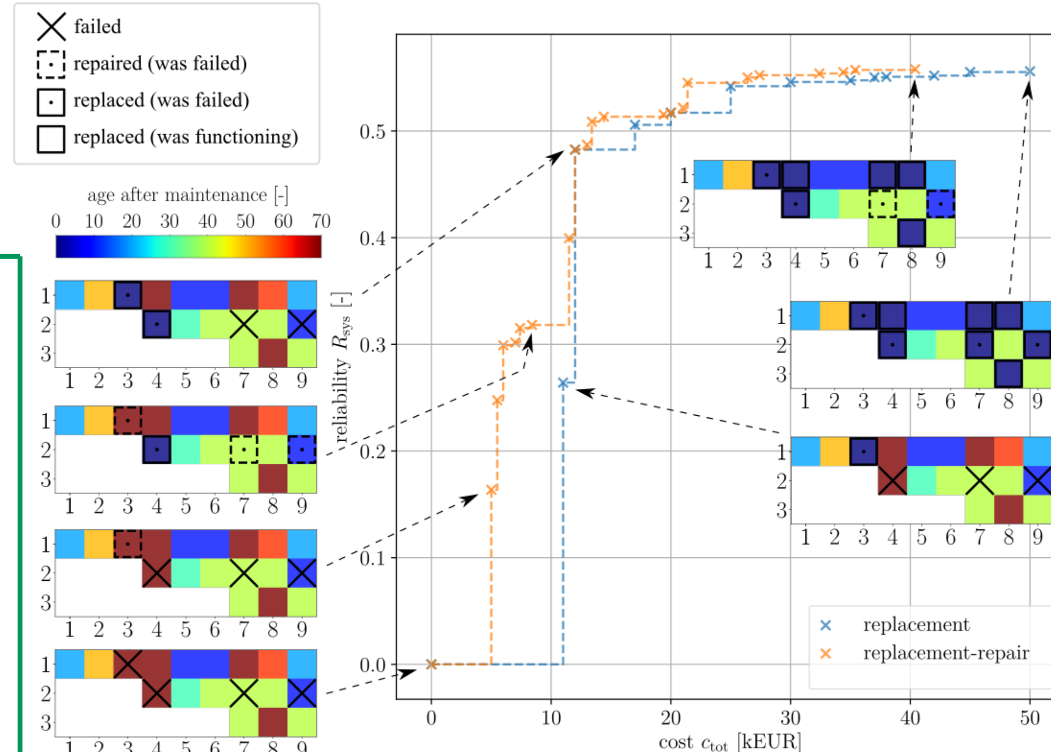
Main modifications to replacement-<u>repair</u> model:

$$R'_k = 1 - \prod_{j \in J_k} (1 - R^0_{k,j} - \Delta R^{y}_{k,j} y_{k,j} - \underline{\Delta R^{x}_{k,j} x_{k,j}}), \quad k \in K$$

Two additional constraints:

$$\begin{cases} y_{k,j} + x_{k,j} \le 1, & k \in K, j \in J_k \\ F_{k,j} + x_{k,j} \le 1, & k \in K, j \in J_k. \end{cases}$$

$R^{\mathbf{x}}_{k,j}$	Reliability of component (k,j) if repaired
$\Delta R^{\mathbf{x}}_{k,j}$	$R^{\mathbf{x}}_{k,j} - R^0_{k,j}$ (improvement in reliability)
$x_{k,j}$	Binary variable defining if component (k,j) is repaired
$F_{k,j}$	Binary parameter defining if component (k,j) is failed



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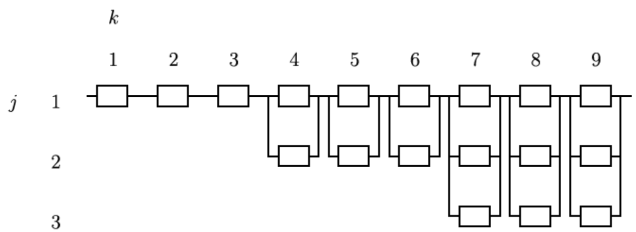
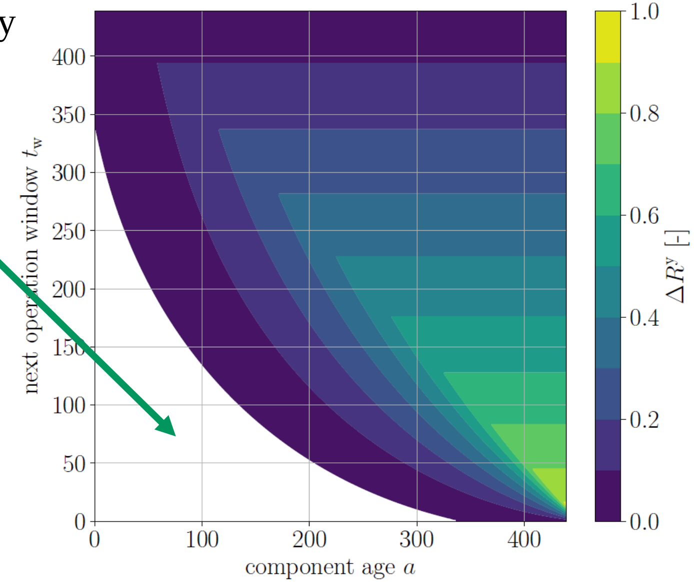
Variable preassigment



The change in reliability is negative
→ replacement is not sensible

Jiang (2013) @ Meeker & Escobar (1998)



$$y_{k,j} = 0, \quad \forall k, j \in \{(k,j) | \Delta R^{y}_{k,j} \leq 0\}$$

$y_{k,j}$ Binary variable defining whether unit j at stage k is replaced

$\Delta R^{x}_{k,j}$ Improvement in the reliability of the unit j at stage k if the component is replaced

Convexified replacement model

Sets

K	Set of stages
J_k	Set of parallel components in stage k

Parameters

$R^0_{k,j}$	Reliability of component (k,j) if not replaced
$R^y_{k,j}$	Reliability of component (k,j) if replaced
$\Delta R^y_{k,j}$	$R^y_{k,j} - R^0_{k,j}$ (improvement in reliability)

Variables

R'_k	Reliability of stage k
$y_{k,j}$	Binary variable defining if component (k,j) is replaced
p	Number of maintenance personnel involved in the maintenance operations

Original formulation:

$$R_{\text{sys}} = \prod_{k \in K} R'_k$$

Multilinear term – non-convex optimization model

$$R'_k = 1 - \prod_{j \in J_k} (1 - R^0_{k,j} - \Delta R^y_{k,j} y_{k,j}), \quad k \in K$$

Convexified replacement model

Sets

K	Set of stages
J_k	Set of parallel components in stage k
$S_{k,m}$	The m^{th} subset of J_k

Parameters

$R^0_{k,j}$	Reliability of component (k,j) if not replaced
$R^y_{k,j}$	Reliability of component (k,j) if replaced
$\Delta R^y_{k,j}$	$R^y_{k,j} - R^0_{k,j}$ (improvement in reliability)

Variables

R'_k	Reliability of stage k
$y_{k,j}$	Binary variable defining if component (k,j) is replaced
p	Number of maintenance personnel involved in the maintenance operations
$\tilde{R}_{\text{sys}}$	$\ln R_{\text{sys}}$
$z_{k,m}$	An additional binary variable

Original formulation:

$$R_{\text{sys}} = \prod_{k \in K} R'_k$$

Multilinear term – non-convex optimization model

$$R'_k = 1 - \prod_{j \in J_k} (1 - R^0_{k,j} - \Delta R^y_{k,j} y_{k,j}), \quad k \in K$$

Convexified formulation:

This is the new objective

$$\tilde{R}_{\text{sys}} \leq \sum_{k \in K} \ln R'_k$$

$$R'_k = 1 - \sum_{S_{k,m} \in \mathbb{S}_k} \Big(z_{k,m} \prod_{j \in S_{k,m}} - \Delta R^y_{k,j} \prod_{j \in J_k \setminus S_{k,m}} (1 - R^0_{k,j}) \Big), \quad k \in K$$

constraints linking $y_{k,j}$ and $z_{k,m}$

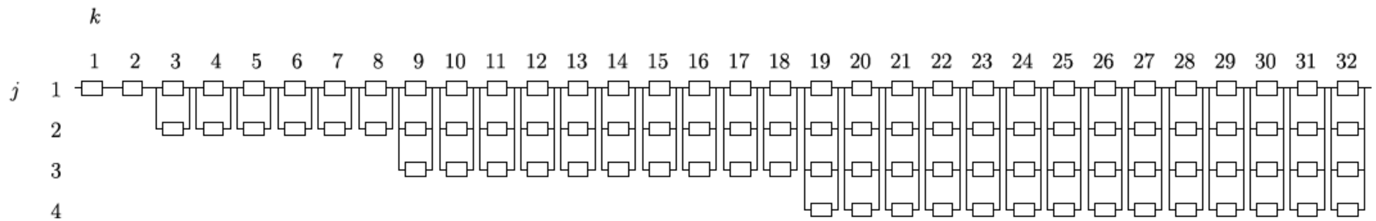
Convexification & Solvers

- **We also convexify the replacement-repair model**
- **We solve the original and convexified models by the following solvers:**

Model	Abbreviation	Solver	Solver type
Non-Convex Replacement model	NCR	BARON 18.5.8	global
Convex Replacement model	CR	DICOPT 2 with CLPEX 12.8.0.0	non-global
Non-Convex Replacement-Repair model	NCRR	BARON 18.5.8	global
Convex Replacement model-Repair model	CRR	DICOPT 2 with CLPEX 12.8.0.0	non-global

Large-scale optimization problems

Basic arrangement containing 100 components



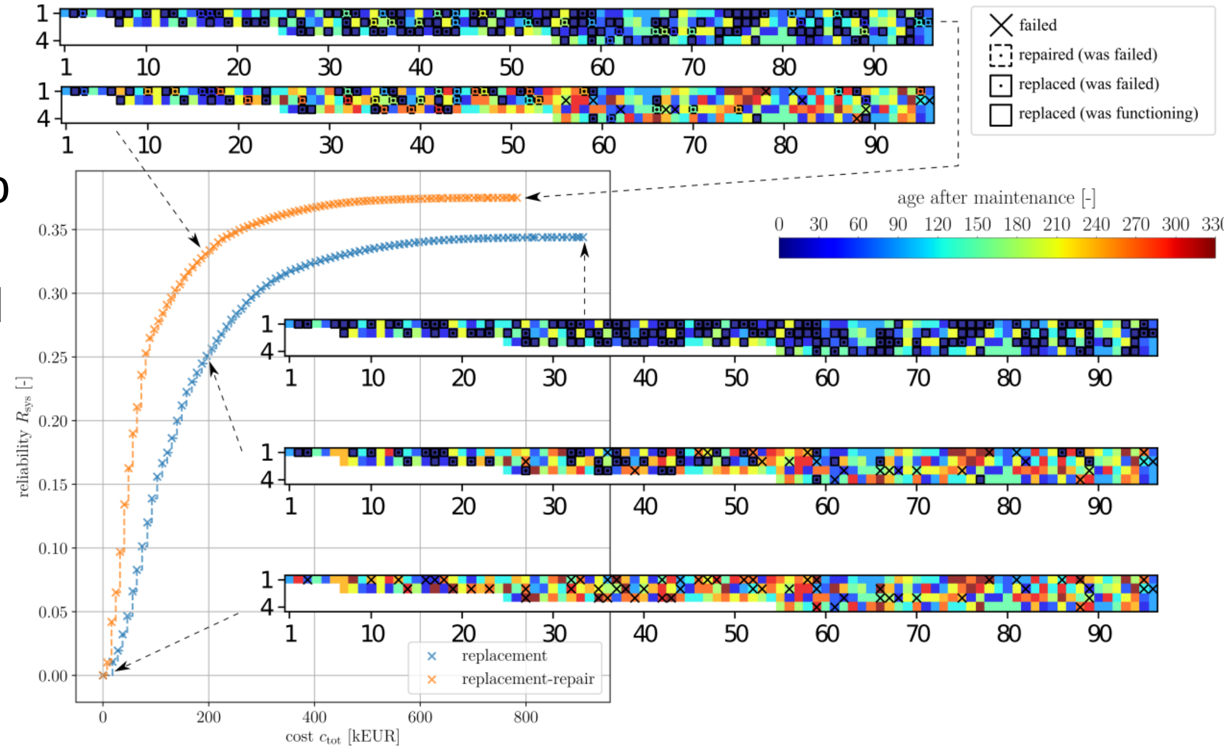
- **1 to 10 basic arrangements in series**
  → Optimization problems with 100 to 1000 components
- **Components are randomly drawn from the catalog**
- **Component ages randomly drawn from the range of {30, 60…330}**
- **20% of the components are failed prior to the maintenance**

Component catalog

Component type	I	II	III	IV	V	VI	VII	VIII	IX	X
cost of replacement c^y [kEUR]	1	3	5	7	8	2	5.5	7.5	10	12
cost of repair c^x [kEUR]	0.5	0.3	1.4	1	2	1.5	2	1	6	4
duration of replacement t^y [h]	30	10	5	7	8	5	7	11	8	12
duration of repair t^x [h]	20	5	2	5	3	9	2	5	15	6

Large-scale example

- **300 components**
- ***ε*-constraint method**
  - The objective is to Maximize R_{sys}
  - c_{tot} is constrained
    - The constraint is iteratively relaxed
    - 100 increments



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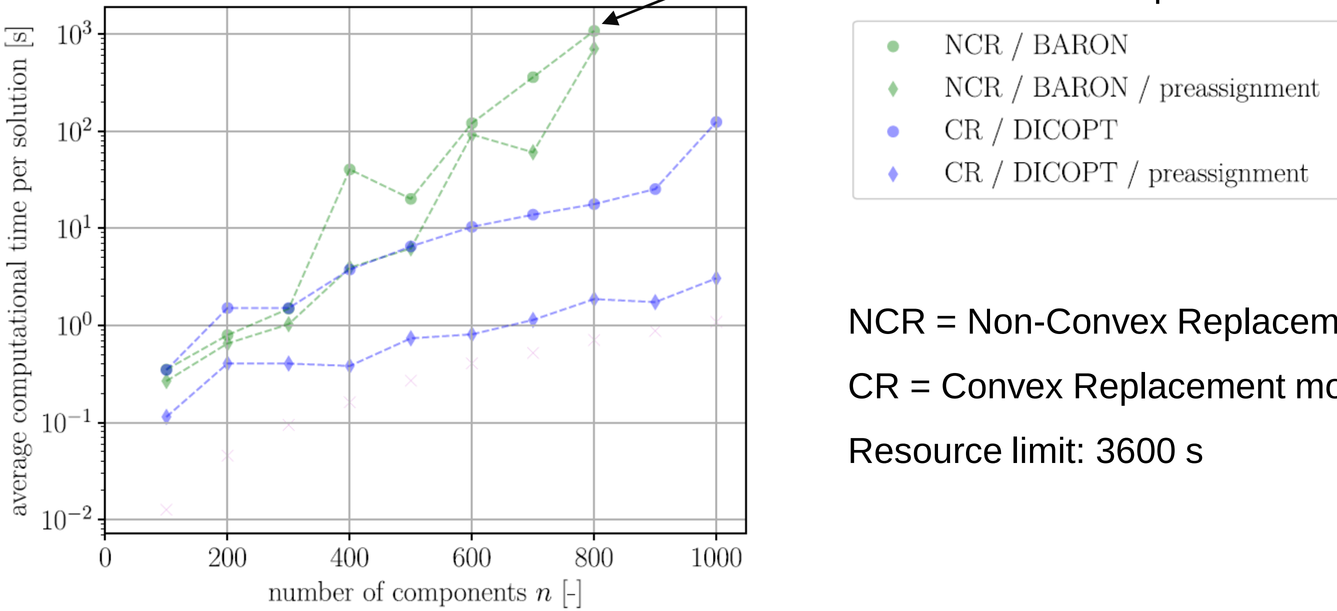
Results: replacement models

A number of runs are timed out
– excluded from the comparison



The optimized results are, on average, within 0.051%

NCR = Non-Convex Replacement model

CR = Convex Replacement model

Resource limit: 3600 s

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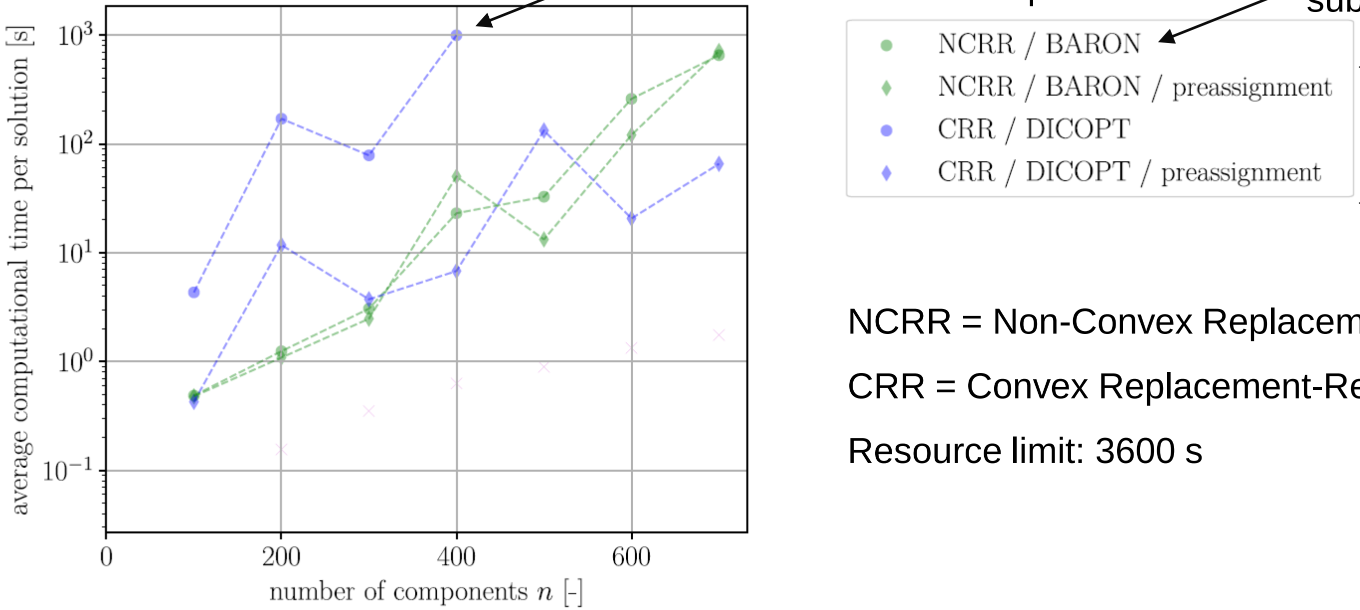
Results: replacement-repair models



A number of runs are timed out – excluded from the comparison

On average, 0.622% sub-optimal

The optimized results are, on average, within 0.042%

NCRR = Non-Convex Replacement-Repair model

CRR = Convex Replacement-Repair model

Resource limit: 3600 s

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Conclusions

- **Linkage between component lifetime data and selective maintenance optimization**
- **If the failure rate is bathtub-shaped, the computation efficiency can be improved by variable preassignments**
  - In our experiments, the improvement was around an order of magnitude
- **Convexifying the <u>replacement model</u> improves the efficiency**
  - The efficiencies of solving the non-convex and convex <u>replacement-repair models</u> seem similar
- **We expand the problem sizes studied in the literature**
  - One maintenance action: 200 → 1000 components
  - Two maintenance actions: 25 → 700 components
  - Assumption: failure rates are bathtub-shaped

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