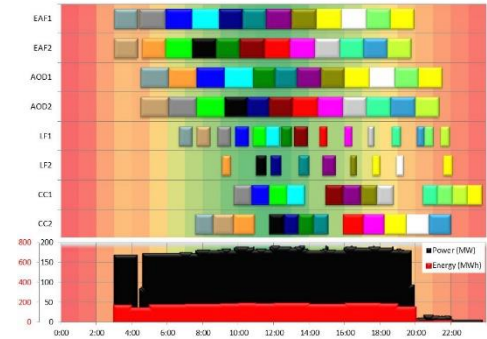


$$\begin{aligned}
 & \left[\begin{array}{l} A_{t,m,tp} \\ T_{s,t,m} \geq cp_{tp}^L \\ T_{s,t,m} \leq cp_{tp}^U \\ T_{e,t,m} \geq cp_{tp}^L \\ T_{e,t,m} \leq cp_{tp}^U \\ \Delta T_{t,m,tp} = P_{t,m} \end{array} \right] \bigvee \left[\begin{array}{l} B_{t,m,tp} \\ T_{s,t,m} \leq cp_{tp}^L \\ (T_{s,t,m} < cp_{tp}^L) \\ T_{e,t,m} \geq cp_{tp}^L \\ T_{e,t,m} \leq cp_{tp}^U \\ \Delta T_{t,m,tp} = T_{e,t,m} - cp_{tp}^L \end{array} \right] \bigvee \\
 & \left[\begin{array}{l} C_{t,m,tp} \\ T_{s,t,m} \geq cp_{tp}^L \\ T_{s,t,m} \leq cp_{tp}^U \\ (T_{e,t,m} > cp_{tp}^L) \\ T_{e,t,m} \geq cp_{tp}^L \\ \Delta T_{t,m,tp} = cp_{tp}^U - T_{s,t,m} \end{array} \right] \bigvee \left[\begin{array}{l} D_{t,m,tp} \\ T_{s,t,m} \leq cp_{tp}^L \\ (T_{s,t,m} \leq cp_{tp}^L) \\ T_{e,t,m} > cp_{tp}^L \\ T_{e,t,m} \geq cp_{tp}^U \\ \Delta T_{t,m,tp} = cp_{tp}^U - cp_{tp}^L \end{array} \right] \bigvee \\
 & \left[\begin{array}{l} E_{t,m,tp} \\ (T_{s,t,m} < cp_{tp}^L) \\ (T_{s,t,m} \leq cp_{tp}^L) \\ T_{e,t,m} \leq cp_{tp}^L \\ (T_{e,t,m} < cp_{tp}^L) \\ \Delta T_{t,m,tp} = 0 \end{array} \right] \bigvee \left[\begin{array}{l} F_{t,m,tp} \\ (T_{s,t,m} > cp_{tp}^L) \\ T_{s,t,m} \geq cp_{tp}^L \\ (T_{e,t,m} \geq cp_{tp}^L) \\ T_{e,t,m} > cp_{tp}^L \\ \Delta T_{t,m,tp} = 0 \end{array} \right] \quad \forall t,m,tp
 \end{aligned}$$



Optimization Models for Scheduling in the Process Industries

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<https://publons.com/researcher/476310/pedro-m-castro/>

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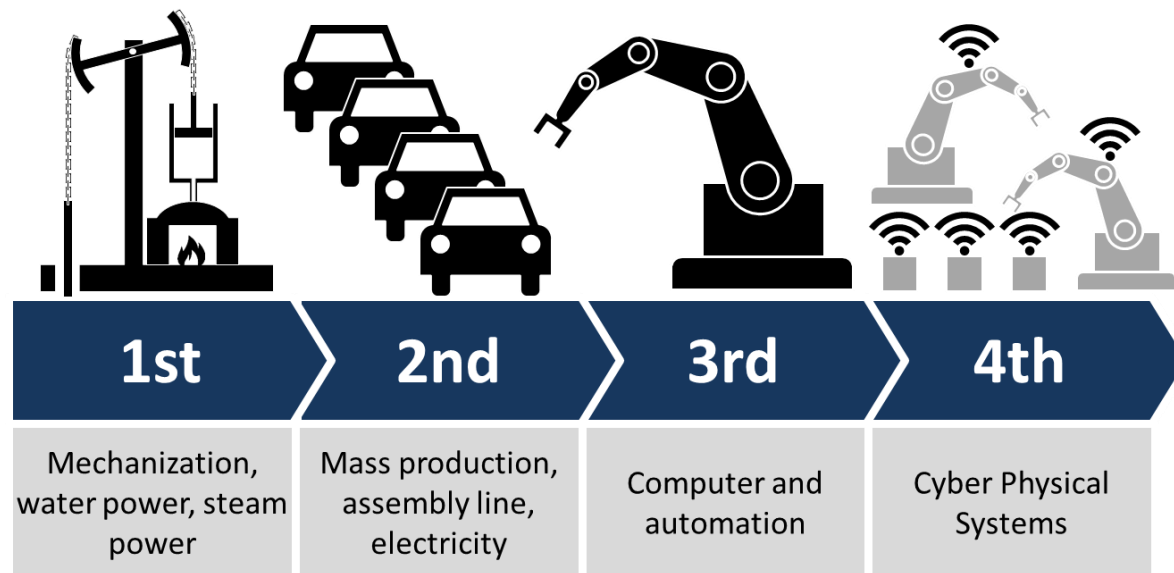
UNIVERSIDADE
DE LISBOA

CMAFCIO

Fourth Industrial Revolution (Industry 4.0)



- **Advanced manufacturing and smart industries**
 - Computer-based **decision-making tools** that enhance system performance
 - **Models that mimic the behavior of a physical system**
 - Quickly exchange data and information with the different systems of the enterprise



Enterprise-Wide Optimization (Grossmann, '05)



- Optimizing the supply, manufacturing & distribution activities of a company to reduce costs & inventories, & to maximize profits, asset utilization & responsiveness
 - Process Systems Engineering
 - Interface of Chemical Engineering & Operations Research
- Scheduling: key element of EWO
 - MILP & MINLP models

Petroleum Supply Chain



Wellhead



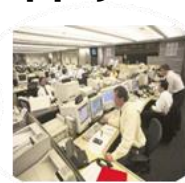
Trading



Transfer
Crude



Refinery
Processing



Planning &
Scheduling



Transfer
Products

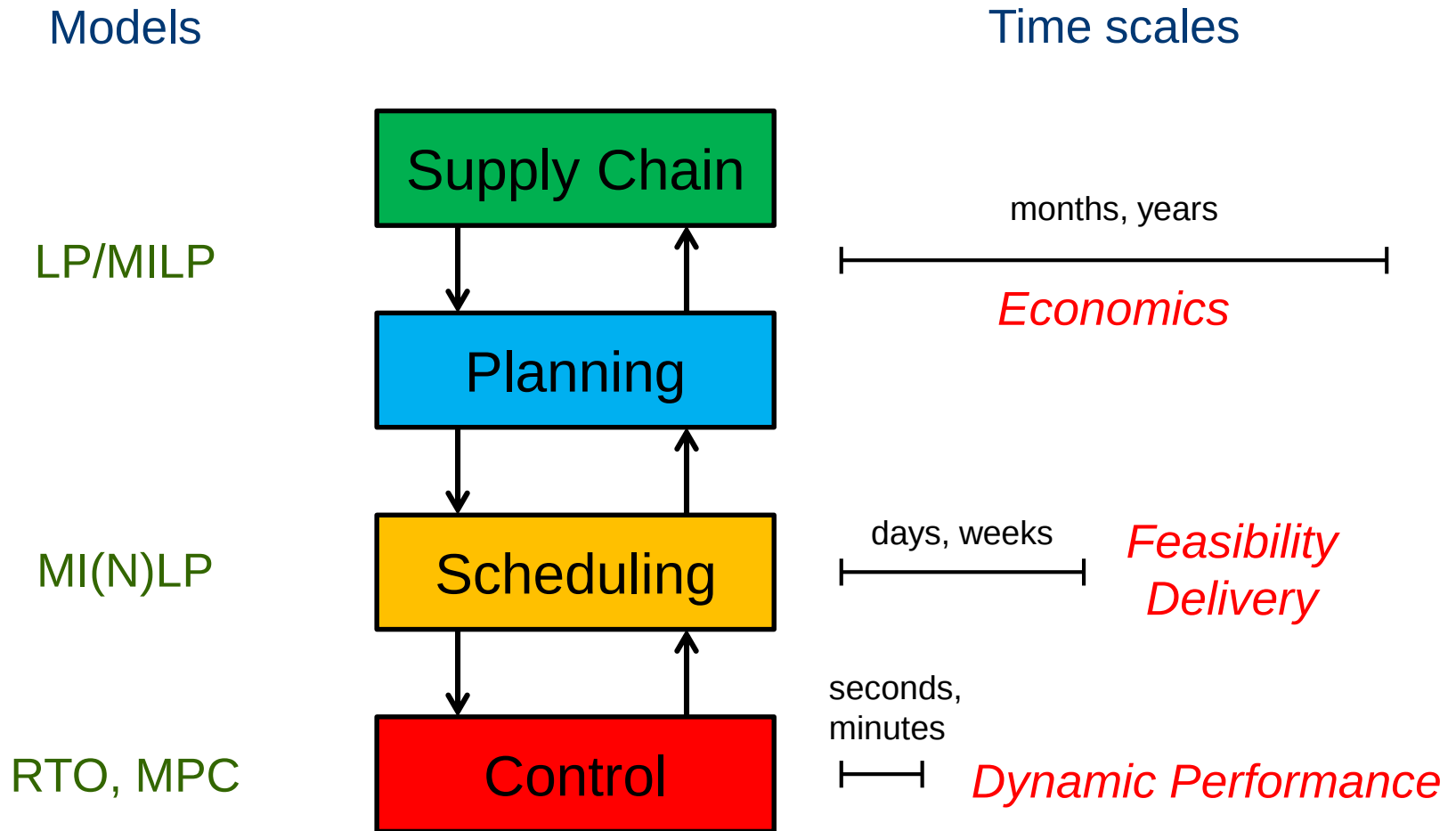


Terminal
Loading



Pump

Optimal operation of manufacturing plants



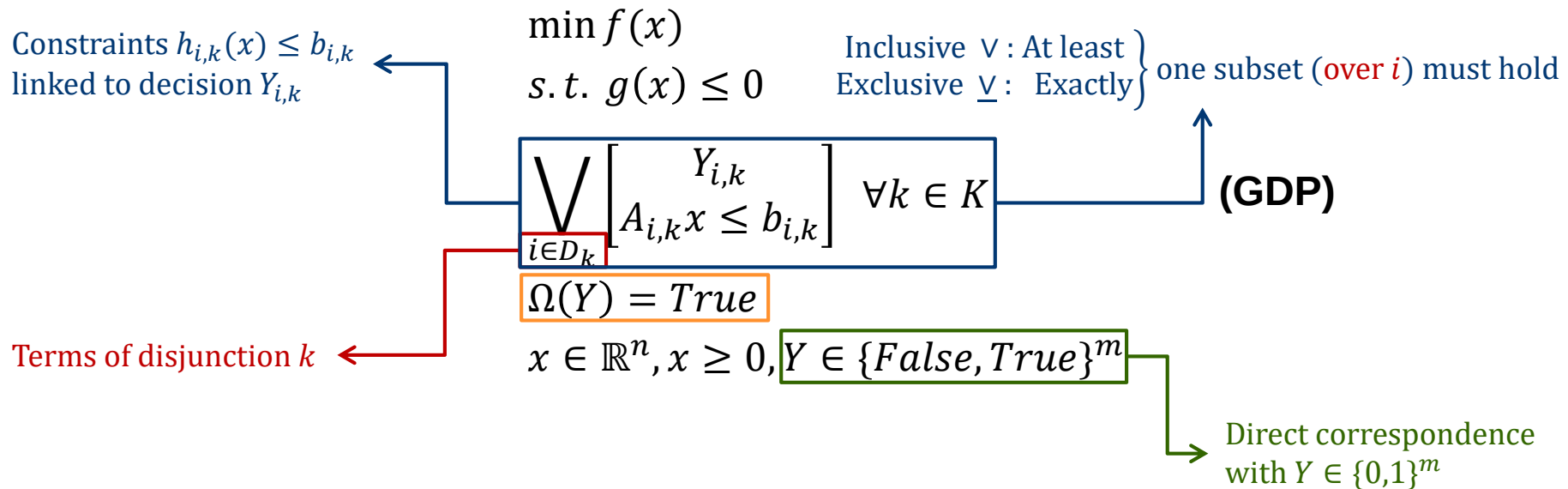
- **Basic concepts**
 - Generalized Disjunctive Programming
 - State- & Resource-Task Network
 - Time representation
- **Industrial case studies**
 - Demand-side management of a steel plant
 - Maintenance scheduling of a power plant
 - Heat integration of a vegetable oil refinery
 - Transportation of refined petroleum products by pipeline
 - Blending problems in petroleum refineries
- **Conclusions**

BASIC CONCEPTS

Generalized Disjunctive Programming



- High-level construct for generating MI(N)LPs
(Balas, '79; Raman & Grossmann, '94; Castro & Grossmann, '12)
 - Constraints with just continuous variables $g(x)$
 - E.g. resource balances, capacity constraints
 - With just binary variables $\Leftarrow$ Logic propositions $\Omega(Y)$
 - Binary & continuous $\Leftarrow$ Disjunctions



Reformulation of linear GDP



- **Disjunctions (Balas, '85)**

- Big-M

- Simplest form but yields poor relaxations

$$A_{i,k}x \leq b_{i,k} + M_{i,k}(1 - Y_{i,k}) \quad \forall k \in K, i \in D_k$$

$$M_{i,k} = \max\{A_{i,k}x - b_{i,k} : 0 \leq x^L \leq x \leq x^U\} \quad \forall k \in K, i \in D_k$$

Tightest big-M parameters

- Convex hull

- At least as tight as big-M, larger problem size and much harder to derive

$$A_{i,k}\hat{x}_{i,k} \leq b_{i,k}Y_{i,k} \quad \forall k \in K, i \in D_k$$

Tighter bounds

$$\hat{x}_{i,k}^L Y_{i,k} \leq \hat{x}_{i,k} \leq \hat{x}_{i,k}^U Y_{i,k} \quad \forall k \in K, i \in D_k$$

$$x = \sum_{i \in D_k} \hat{x}_{i,k} \quad \forall k \in K$$

Disaggregated variables
(new set)

- Common constraint

$$\sum_{i \in D_k} Y_{i,k} \begin{cases} \geq 1 & \text{if inclusive OR} \\ = 1 & \text{if exclusive OR} \end{cases} \quad \forall k \in K$$

- **Logic propositions**

- Replace with linear inequalities (Clocksin & Mellish, '81)

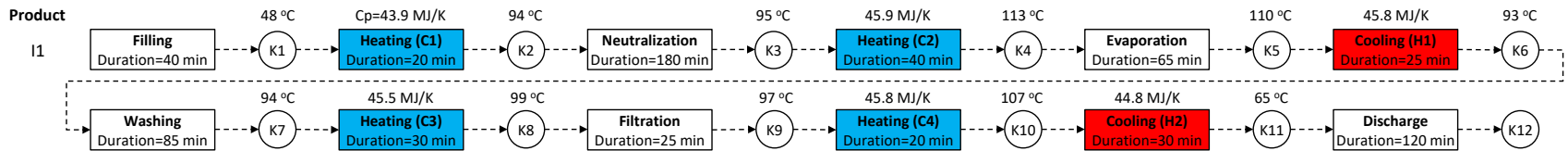
Production recipe & environment



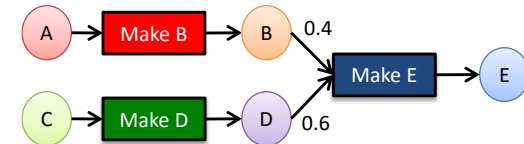
Recipe



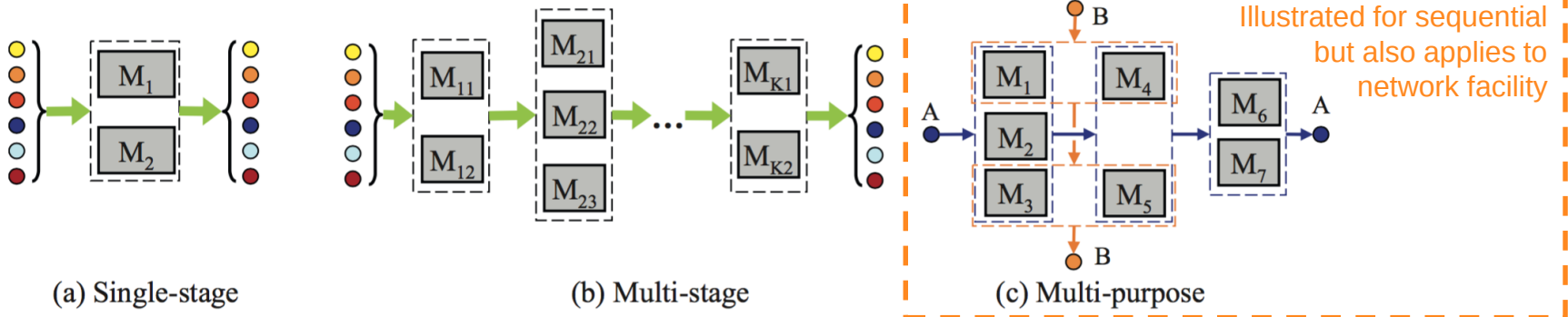
– Sequence of tasks with known duration/processing rate



- Need to consider multiple materials?
 - No: Identity is preserved $\Rightarrow$ sequential facility
 - Yes: Material-based $\Rightarrow$ network facility



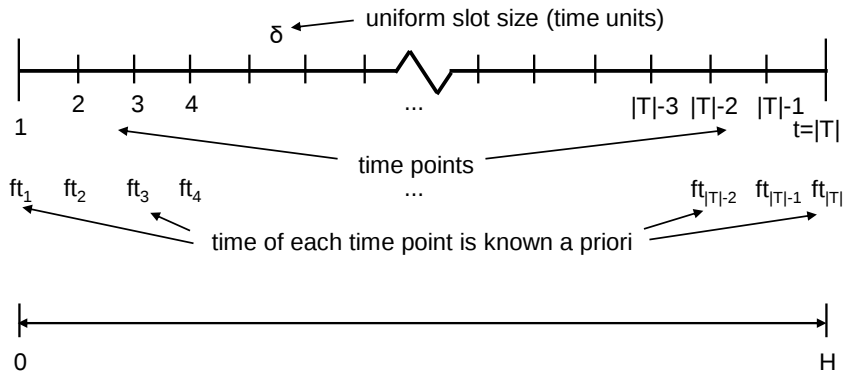
Environment



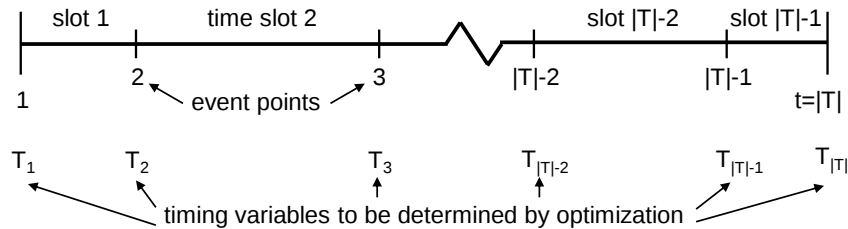
Time representation



• Discrete time



• Continuous time



- Single time grid for all resources
- Multiple time grids

• Precedence

– General

$$\boxed{i} \quad \boxed{i'} \quad \underline{\vee} \quad \boxed{i'} \quad \boxed{i} \quad \forall i < i'$$

$$Y_{i,i'} = \text{True} \quad Y_{i,i'} = \text{False}$$

$$\left[x_{i'} \geq x_i + p_i \right] \bigvee \left[x_i \geq x_{i'} + p_{i'} \right] \forall i < i'$$

duration of order i $\uparrow$ $\uparrow$ starting time of order i

– Immediate

$$\boxed{i} \quad \boxed{i'} \quad \underline{\vee} \quad \boxed{i} \quad \boxed{i''} \quad \underline{\vee} \quad \boxed{i} \quad \forall i$$

$$Y_{i,i'} = \text{True} \quad Y_{i,i''} = \text{True} \quad Y_i^{\text{last}} = \text{True}$$

$$\underline{\vee} \quad i' \neq i \left[x_{i'} \geq x_i + p_i \right] \bigvee \left[x_i \geq x_{i'} + p_{i'} \right] \forall i' \neq i$$

GDP facilitates modeling of equipment availability constraint

Comparison of representation concepts



- **Discrete-time models are more robust**
 - Excel for batch processes with frequent external events
 - Reducing data accuracy ($\uparrow \delta$)
is a good way to reduce complexity (problem size)
- **Continuous-time models**
 - Better for continuous processes (single grid)
 - Can be orders of magnitude faster (multiple grids)
 - Results for industrial case study (Castro et al., '14)
 - Multistage batch plant with single unit per stage

Time representation	Discrete (single, uniform grid)	Continuous (single grid)	Continuous (multiple time grids)
Time slots (per grid)	564	16	4
Binary variables	31583	3584	79
Total variables	60914	4435	141
Equations	29372	4435	132
Optimal solution	564	571	564
Computational time (CPUs)	217	25252	0.21

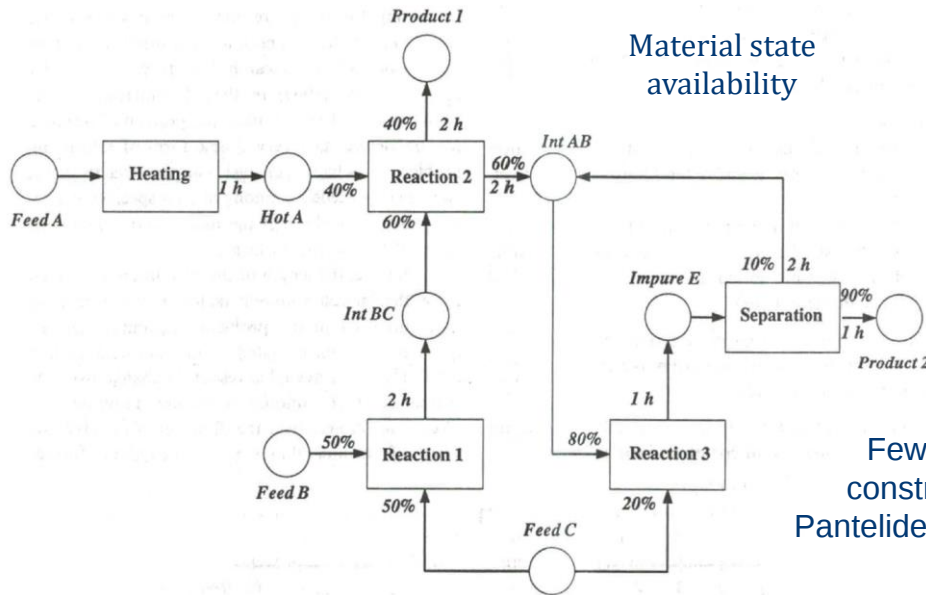
State-Task Network (STN) (Kondili, Pantelides & Sargent '93)



Material balances (multiperiod)

$$[S_{s,t}] = S_{s,t-1} + \sum_i \sum_j \sum_{\theta=0}^{\tau_i} (\bar{\rho}_{i,s,\theta} B_{i,j,t-\theta} - \rho_{i,s,\theta} B_{i,j,t-\theta}) + [R_{s,t} - D_{s,t}] \forall s, t$$

Production Consumption Batch size Raw-material supply & product demand



Material state availability

Equipment allocation constraints

$$\sum_{i'} \sum_{t'=t}^{t+\tau_i-1} W_{i',j,t'} - 1 \leq M(1 - [W_{i,j,t}]) \forall i, j, t$$

Processing time Assigns start of task i to unit j time t

Fewer & tighter constraints (Shah, Pantelides & Sargent '93)



$$\sum_i \sum_{t'=t}^{t-\tau_i+1} W_{i,j,t'} \leq 1 \forall j, t$$

- Process representation model

- Complex recipes, multiple processing routes, shared intermediates, recycles
 - Material states as circles, tasks as rectangles
- Differentiated treatment between material states & equipment units

- One of most important papers in Process Systems Engineering (PSE)

- 700 citations (ISI), #5 of all time in Comp. Chem. Eng.

Resource-Task Network (RTN) (Pantelides '94)



• Generalization of STN

– Tasks

- Rectangles

– Resources (states, units, etc.)

- Circles

– Structural parameters

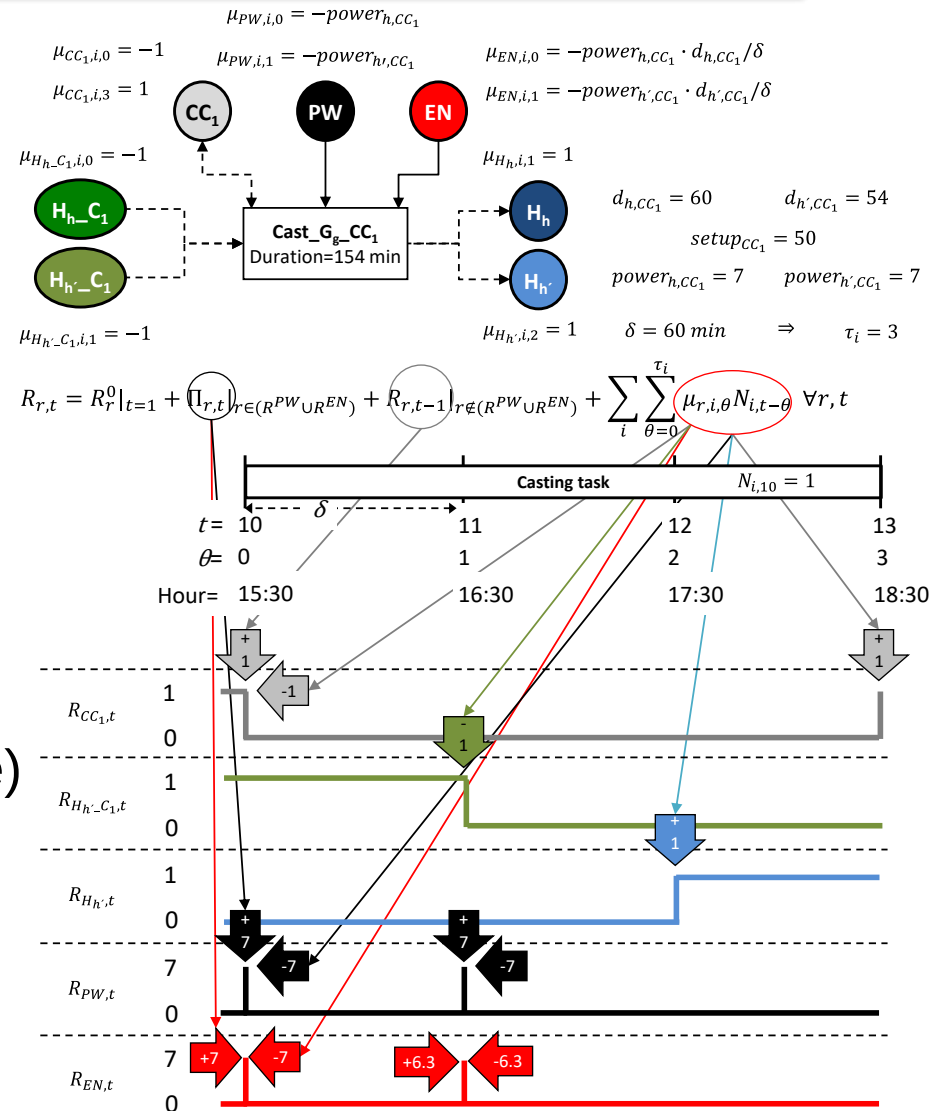
- Link tasks & resources
- May be difficult to find

• RTN mathematical model

– Very simple & tight (discrete-time)

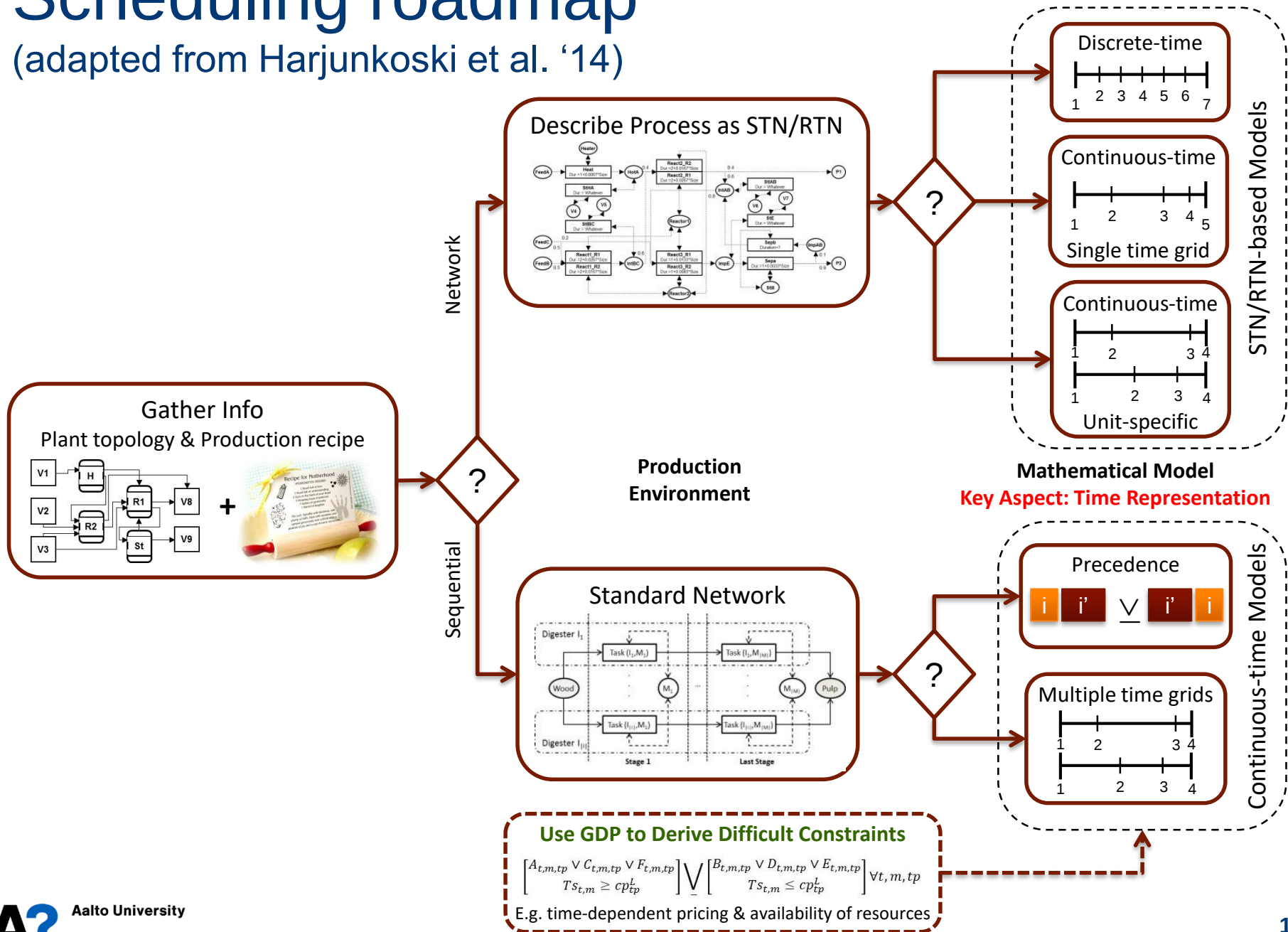
- Few sets of constraints

– Magic is in excess resource balances!



(adapted from Harjunkoski et al. '14)

(adapted from Harjunkoski et al. '14)



STEEL INDUSTRY



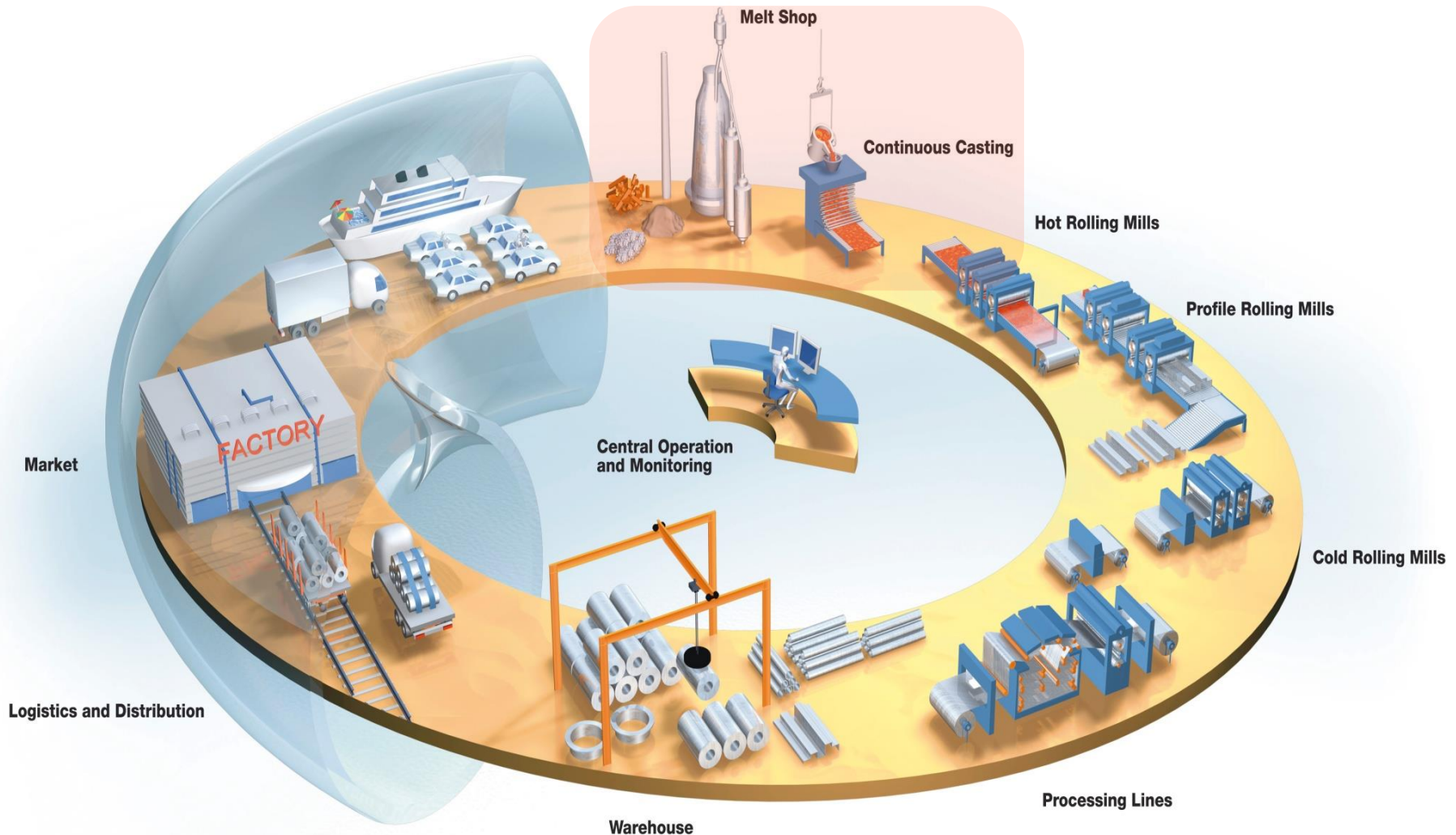
Resource-task network formulations for industrial demand side management of a steel plant.

[Ind. Eng. Chem. Res. 2013, 52, 13046–13058](#)

Steel Supply Chain



Most energy intensive step



Steelmaking process

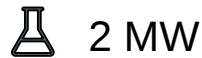


- 4 stages, 2 units in parallel

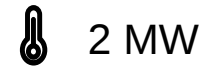
Electricity costs
50-100 M€/year



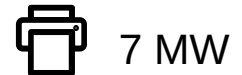
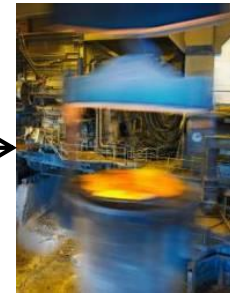
Electric Arc Furnace:
melts scrap steel



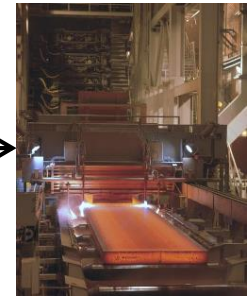
Argon Oxygen Decarburization:
adjusts the chemistry



Ladle Furnace:
adjusts the temperature

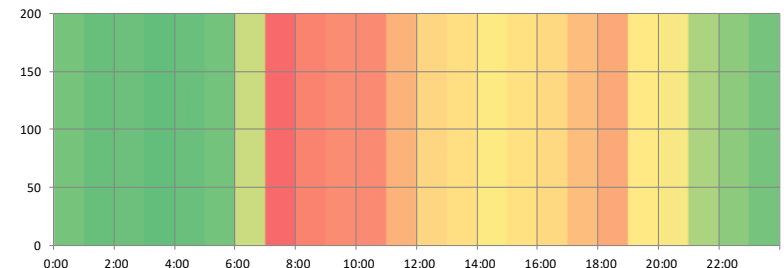


Continuous Casting:
creates steel slabs



- Industrial Demand Side Management**

- Electricity provider gives economic incentives to industry to alter their electricity usage behavior
- Price-based approach
(**epexspot** day-ahead market)
 - Customers switch production to low-priced periods

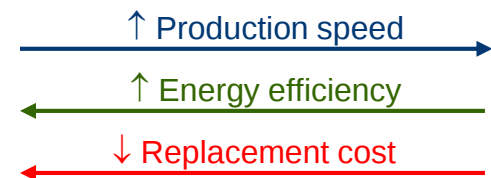


EAFFs have multiple operating modes



- Flexibility to select power mode for a steel heat
- Decision not easy due to tradeoff between:
 - Production speed (👍 more tasks in a low-cost period)
 - Energy efficiency
 - Electrode replacement frequency
 - Energy and maintenance costs are comparable

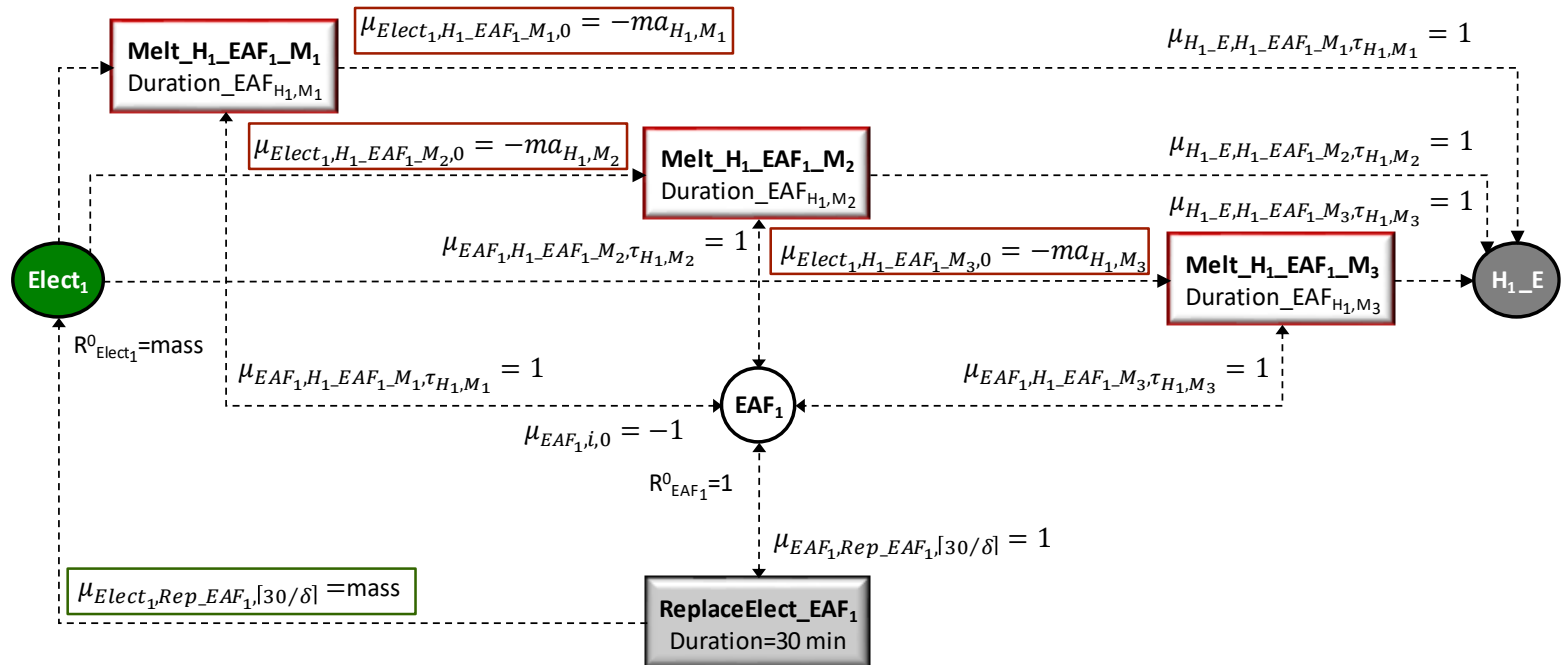
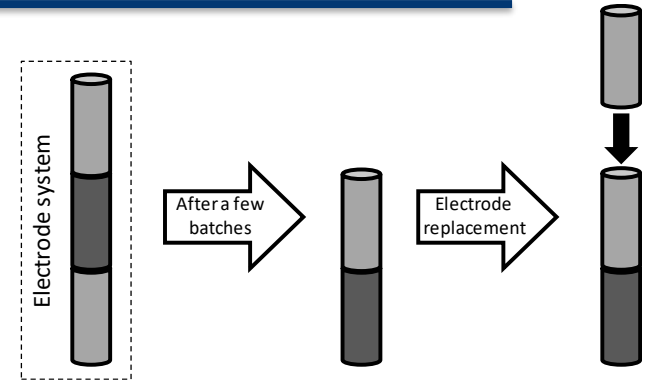
Operating mode m	M_1	M_2	M_3
Power consumption $pw_{k=1,m}$ (MW)	40	60	75
Duration for steel heats $H_1-H_8, H_{13}-H_{17}, H_{21}-H_{24}$ (min)	69	49	41
Duration for steel heats $H_9-H_{12}, H_{18}-H_{20}$ (min)	76	54	45
Electrode mass consumption $ma_{h,m}$ for $H_1-H_8, H_{13}-H_{17}, H_{21}-H_{24}$ (kg)	123.3	131.4	137.4
Electrode mass consumption $ma_{h,m}$ for $H_9-H_{12}, H_{18}-H_{20}$ (kg)	135.7	144.5	151.2



Modelling electrode degradation



- Mass consumed during melting
- Need to replace electrode after a few batches
 - How many? Depends on **modes selected**
 - Force execution of replacement task when **electrode mass** becomes negative
 - Regenerates electrode to allow EAF to resume operation



Optimal schedule (discrete-time, $\delta=10$ min)

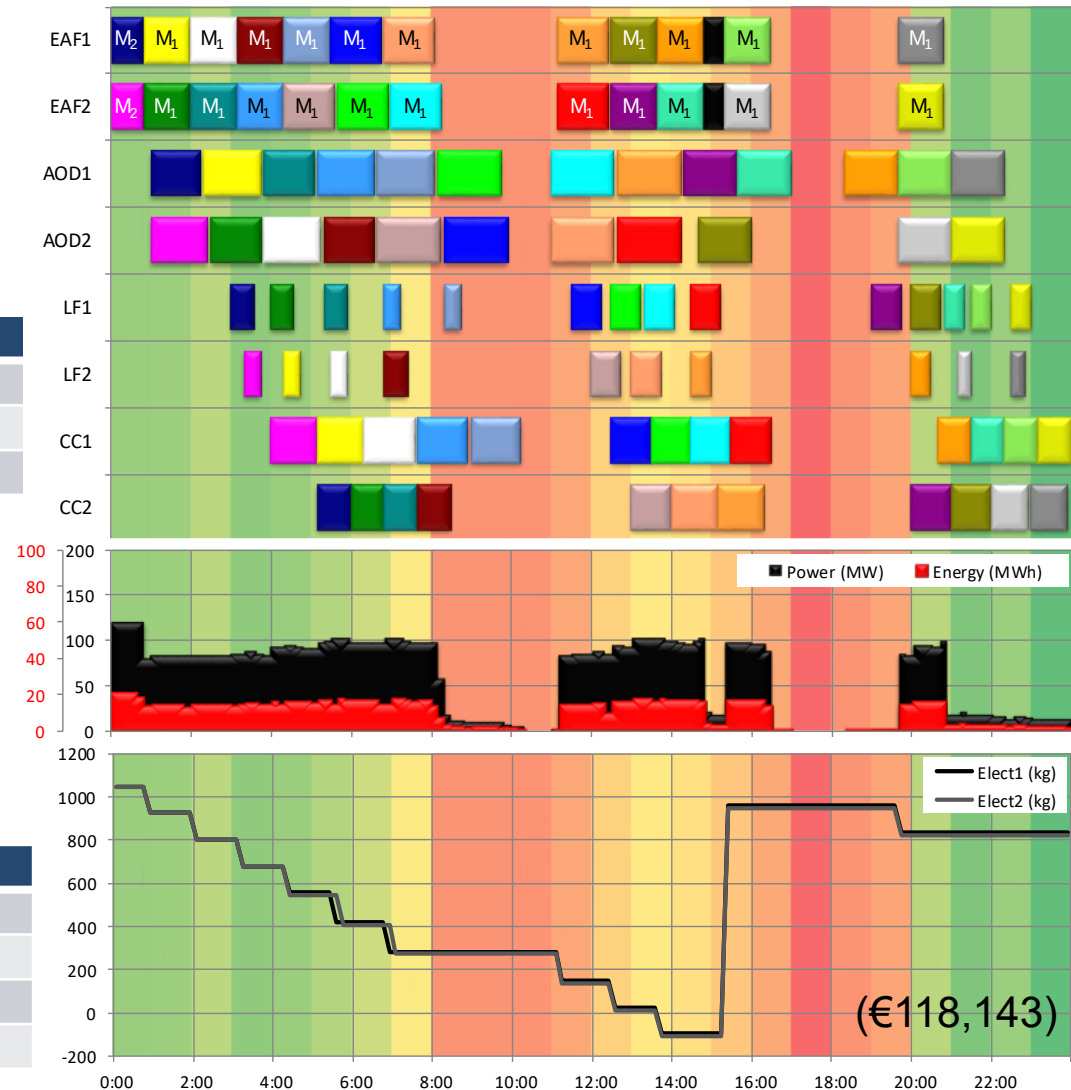


- # heats in (M_1, M_2, M_3)
– (22,2,0)
- How does it compare to single mode operation?
– Same to process all heats in M_1

Mode	Cost (€)	Increase
M_1	118,146	0.00%
M_2	122,089	3.34%
M_3	126,675	7.22%

- Results change with electricity price profile
– 2x average price: 102 €/MWh
– # heats: (12,9,3)
– Best single mode is now M_2

Mode	Cost (€)	Increase
(M_1, M_2, M_3)	174,103	-
M_1	180,646	3.76%
M_2	174,417	0.18%
M_3	186,436	7.08%



POWER PLANT OF A CHEMICAL COMPLEX



Optimal maintenance scheduling of a gas engine power plant using generalized disjunctive programming.

[AIChE J. 2014, 60, 2083–2097](#)

Generator maintenance of a power plant



Sasolburg Gas Engine Power Plant
 18 Wärtsilä 34 SG engines (140 MW)
 Largest plant of its kind in Africa
 Highly efficient and environmentally friendly facility
 Inaugurated July 2013

Sasolburg Chemical Complex



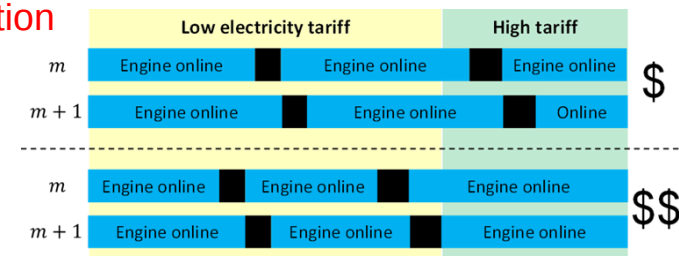
Excess
production



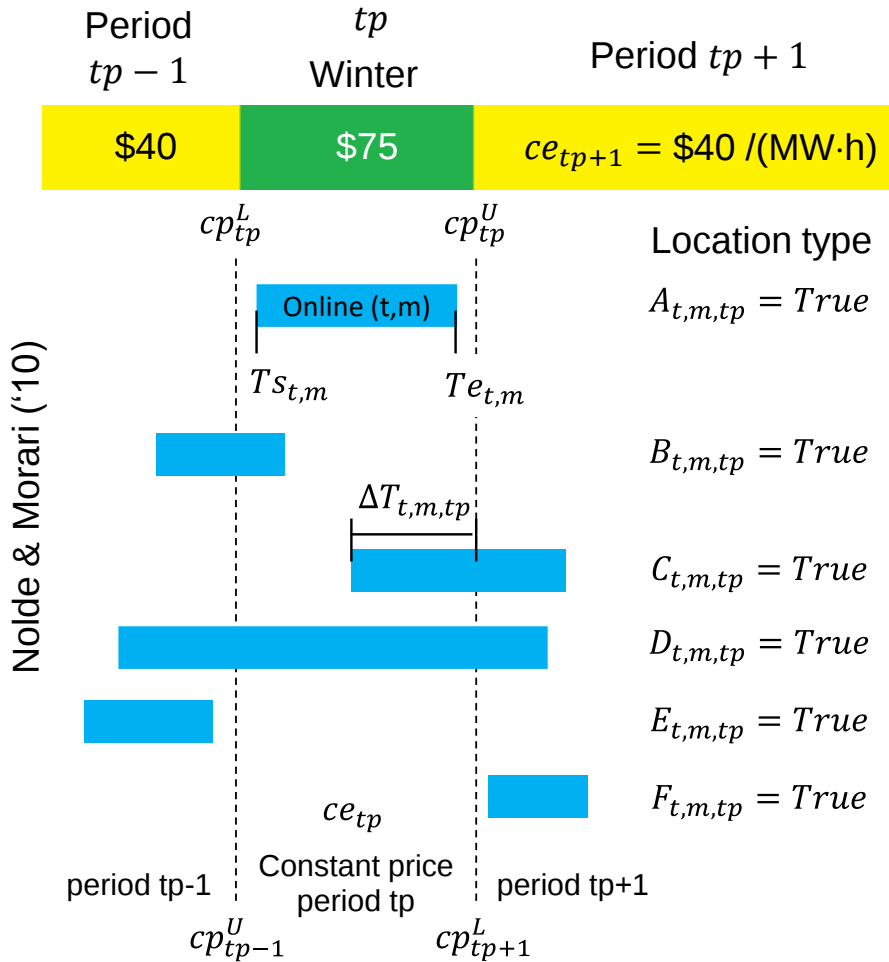
National Grid

- **Goal: Maintain engines when electricity is cheaper (seasonal prices)**
 - Perform preventive maintenance after a number of hours online
 - Large discrepancy in times $\Rightarrow$ **discrete-time not an option**

Operating slot t	1	2	3	4	5	6	7	8	9	...
Max online [h]				2500				3000	2500	...
Min online [h]				2000				2000	2000	...
shutdown [h]	12	72	12	96	12	120	12	432	12	...



Continuous-time GDP model

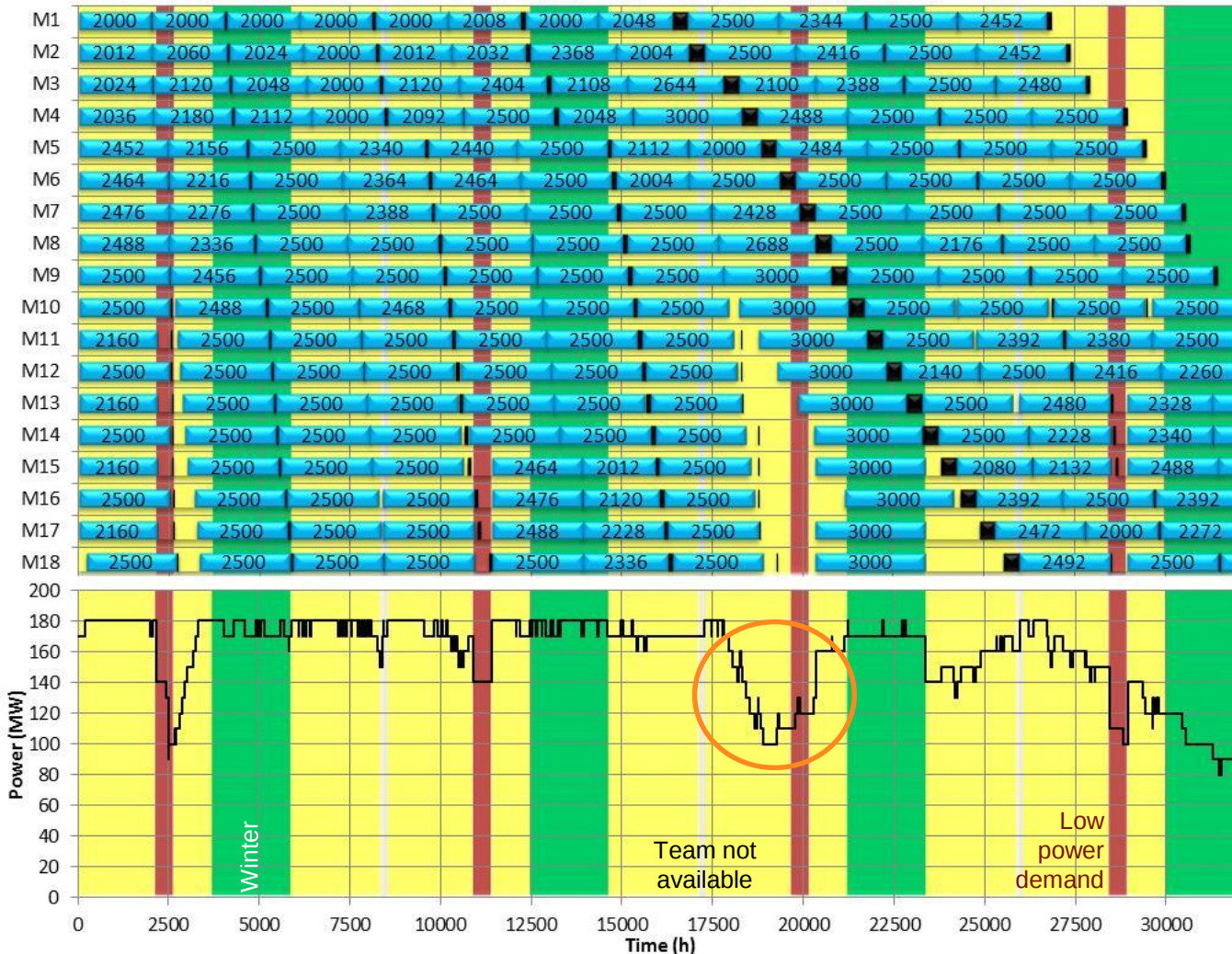


$$\bigvee_- \begin{bmatrix} A_{t,m,tp} \\ TS_{t,m} \geq cp_{tp}^L \\ TS_{t,m} \leq cp_{tp}^U \\ Te_{t,m} \geq cp_{tp}^L \\ Te_{t,m} \leq cp_{tp}^U \\ \Delta T_{t,m,tp} = P_{t,m} \end{bmatrix} \bigvee_- \begin{bmatrix} B_{t,m,tp} \\ TS_{t,m} \leq cp_{tp}^L \\ (TS_{t,m} \leq cp_{tp}^U) \\ Te_{t,m} \geq cp_{tp}^L \\ Te_{t,m} \leq cp_{tp}^U \\ \Delta T_{t,m,tp} = Te_{t,m} - cp_{tp}^L \end{bmatrix} \bigvee_- \begin{bmatrix} C_{t,m,tp} \\ TS_{t,m} \geq cp_{tp}^L \\ TS_{t,m} \leq cp_{tp}^U \\ (Te_{t,m} \geq cp_{tp}^L) \\ Te_{t,m} \geq cp_{tp}^U \\ \Delta T_{t,m,tp} = cp_{tp}^U - TS_{t,m} \end{bmatrix} \bigvee_- \begin{bmatrix} D_{t,m,tp} \\ TS_{t,m} \leq cp_{tp}^L \\ (TS_{t,m} \leq cp_{tp}^U) \\ (Te_{t,m} \geq cp_{tp}^L) \\ Te_{t,m} \geq cp_{tp}^U \\ \Delta T_{t,m,tp} = cp_{tp}^U - cp_{tp}^L \end{bmatrix} \bigvee_- \begin{bmatrix} E_{t,m,tp} \\ (TS_{t,m} \leq cp_{tp}^L) \\ (TS_{t,m} \leq cp_{tp}^U) \\ Te_{t,m} \leq cp_{tp}^L \\ (Te_{t,m} \leq cp_{tp}^U) \\ \Delta T_{t,m,tp} = 0 \end{bmatrix} \bigvee_- \begin{bmatrix} F_{t,m,tp} \\ (TS_{t,m} \geq cp_{tp}^L) \\ TS_{t,m} \geq cp_{tp}^U \\ (Te_{t,m} \geq cp_{tp}^L) \\ (Te_{t,m} \geq cp_{tp}^U) \\ \Delta T_{t,m,tp} = 0 \end{bmatrix} \forall t, m, tp$$

$$A_{t,m,tp} + B_{t,m,tp} + C_{t,m,tp} + D_{t,m,tp} + E_{t,m,tp} + F_{t,m,tp} = 1$$

$$\max \text{Revenue} = \sum_t \sum_m \sum_{tp} \Delta T_{t,m,tp} \cdot ce_{tp} \cdot pw_m$$

Maintenance plan for 3 years



- Engines never idle during winter
- Major bottleneck in year 3
 - Worth to hire additional team?

# slots	Revenue (M\$)	Gap (%)	CPUs
4	83.64	-	0.51
6	127.31	-	6.68
8	163.17	-	345
10	213.85	-	298
12	251.45	1.29	3600

Solution shown

VEGETABLE OIL REFINERY



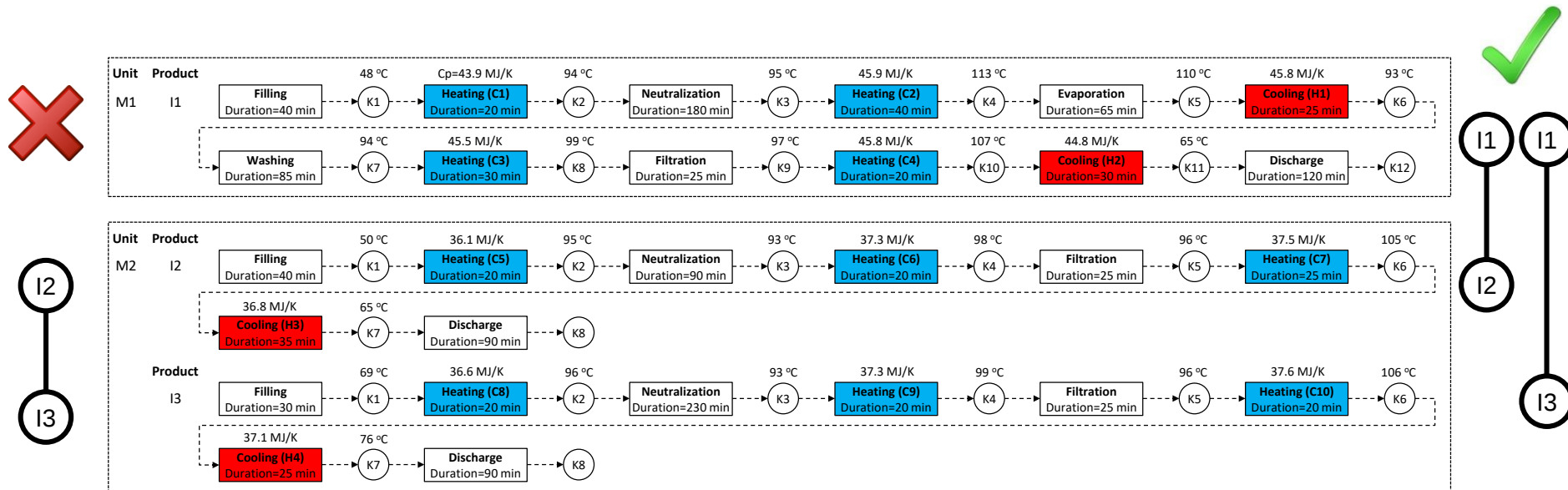
Optimal scheduling of single stage batch plants
with direct heat integration.

CACE 2015, 82, 172-185

Production recipe for batch plant (STN)



- Subset of tasks i requires heating or cooling
 - Heating $\Rightarrow$ Cold task c (stream ps)
 - Cooling $\Rightarrow$ Hot task h (stream ps)
 - Fixed duration, heat capacity cp_{ps} , initial t_{ps}^{in} & final temperatures t_{ps}^{out}
- Current strategy relies on external cold CU & hot HU utilities
- Test heat integration between tasks of different units

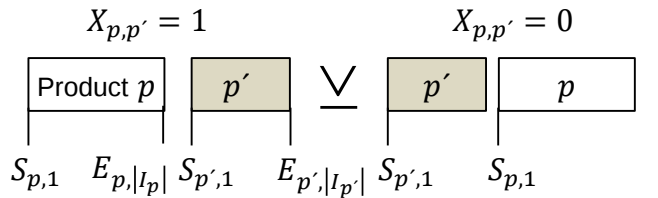


Simultaneous scheduling & heat integration



• Scheduling block

- Finds product sequence
 - General precedence



$$\left[\begin{array}{c} X_{p,p'} \\ E_{p,|I_p|} \leq S_{p',1} \end{array} \right] \bigvee \left[\begin{array}{c} \neg X_{p,p'} \\ E_{p',|I_{p'}|} \leq S_{p,1} \end{array} \right] \forall p, p' > p, m$$

$$E_{p,i} = S_{p,i} + d_{p,i} \quad \forall p, i \in I_p$$

$$S_{p,i+1} = E_{p,i} \quad \forall p, i \in I_p \setminus |I_p|$$

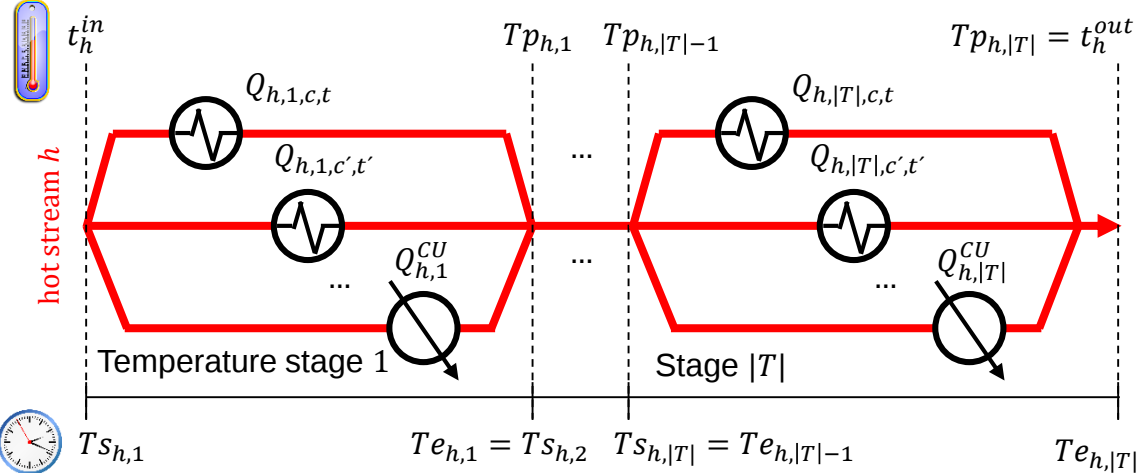
$$MK \geq E_{p,|I_p|} \quad \forall p$$

• Linking constraints

$$TS_{ps,1} = \sum_p \sum_{i:ps \in PS_{p,i}} S_{p,i} \quad \forall ps$$

$$Te_{ps,|T|} = \sum_p \sum_{i:ps \in PS_{p,i}} E_{p,i} \quad \forall ps$$

• Heat integration block



$$\bigvee_t \bigvee_{t'} \left[\begin{array}{c} Y_{h,t,c,t'} \\ TS_{h,t} = TS_{c,t'} \\ Te_{h,t} = Te_{c,t'} \\ Tp_{h,t} \geq Tp_{c,t'} + \Delta t \\ Q_{h,t,c,t'} \leq q_{h,c} \\ Q_{h,t'',c,t'''} = 0 \quad \forall t'' \neq t \vee t''' \neq t' \end{array} \right] \bigvee \left[\begin{array}{c} Y_{h,c}^{no} \\ TS_{h,t} \geq 0 \quad \forall t \\ TS_{c,t'} \geq 0 \quad \forall t' \\ Q_{h,t,c,t'} = 0 \quad \forall t, t' \end{array} \right] \quad \forall h, c$$

$$cp_h(Tp_{h,t-1} - Tp_{h,t}) = \sum_c \sum_{t'} Q_{h,t,c,t'} + Q_{h,t}^{CU} \quad \forall h, t$$

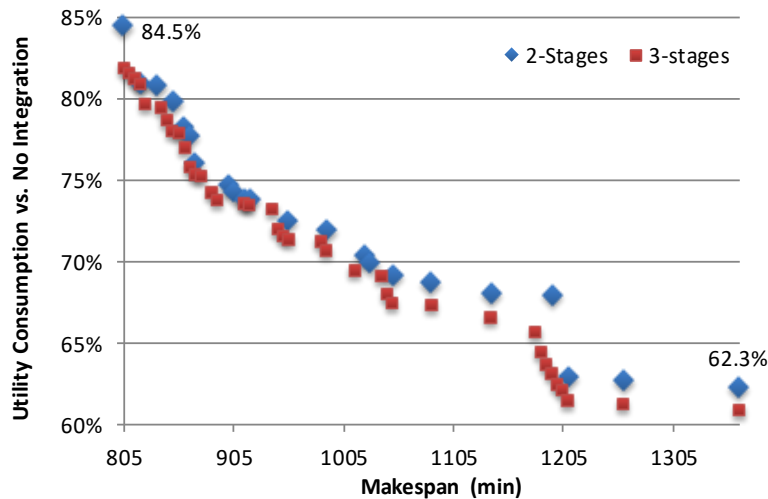
$$cp_c(Tp_{c,t'} - Tp_{c,t'-1}) = \sum_h \sum_t Q_{h,t,c,t'} + Q_{c,t'}^{HU} \quad \forall c, t'$$

$$UT = \sum_h \sum_t Q_{h,t}^{CU} + \sum_c \sum_{t'} Q_{c,t'}^{HU}$$

Tradeoff makespan vs. utility consumption



• 26-streams problem



Energy savings



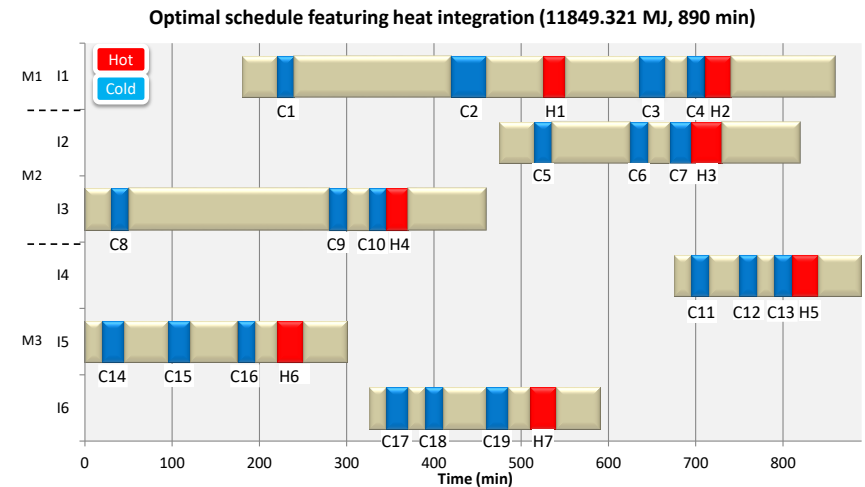
15.5 %



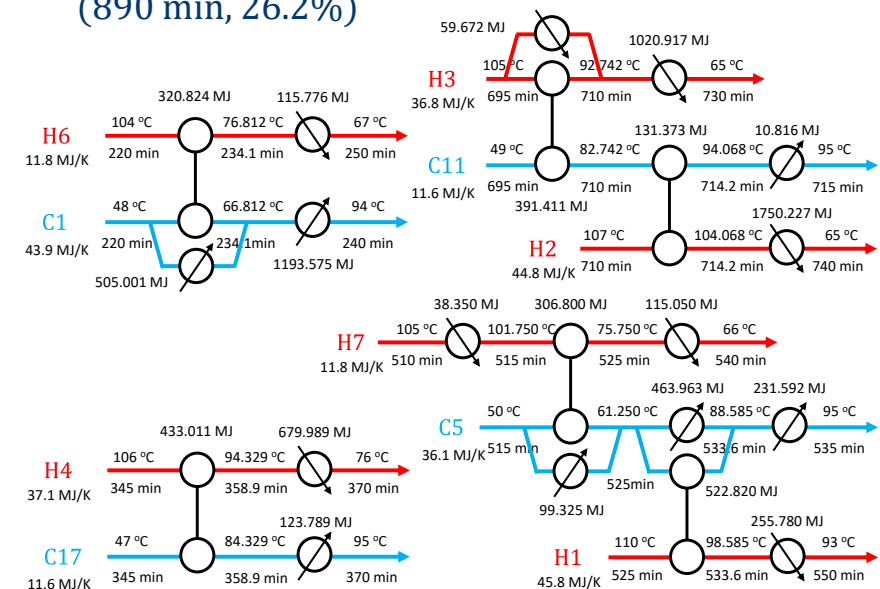
37.7 %

Computational time (CPUs)

Problem/Stages	2	3
18 streams	29	927
26 streams	463	202,652
33 streams	171,971	-



(890 min, 26.2%)



TRANSPORT OF REFINED PETROLEUM PRODUCTS BY PIPELINE



Product-centric continuous-time formulation for pipeline scheduling.

[CACE 2017, 104, 283-295](#)

Batch-centric scheduling formulation for treelike pipeline systems with forbidden product sequences.

[CACE 2019, 122, 2-18](#)

Multiproduct liquid pipelines



- Advantages

- Carry large volumes & multiple products
- Lower carbon footprint vs. trucks & trains

- Aim is to find volumes & timing of injections (from/to input/output nodes)

- Pipeline segments always full, v_s^S (m³)
- Flowrate bounds can vary between segments
 - e.g. $[\rho_s^{S,\min}, \rho_s^{S,\max}]$ (m³/h)

- Modelling challenge

- Track product coordinates in the network
 - To trigger injection/delivery events

Refinery



Distribution Center



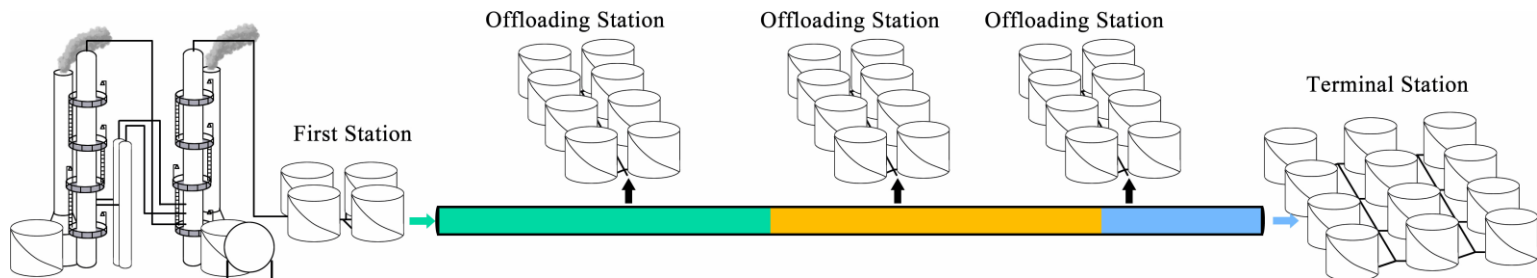
$$v_s^S = 12100 \text{ m}^3$$



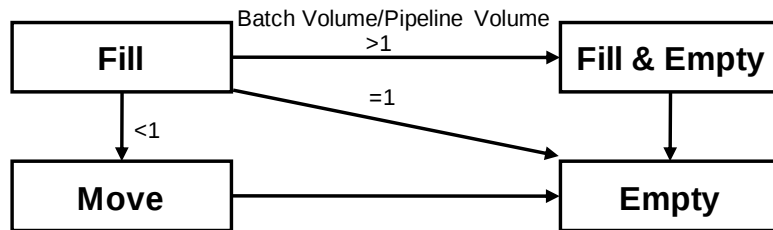
$$LC_{s,p4,t} = 0 \quad RC_{s,p4,t} = 4300$$

$$LC_{s,p6,t} = 4300 \quad RC_{s,p6,t} = 7700$$

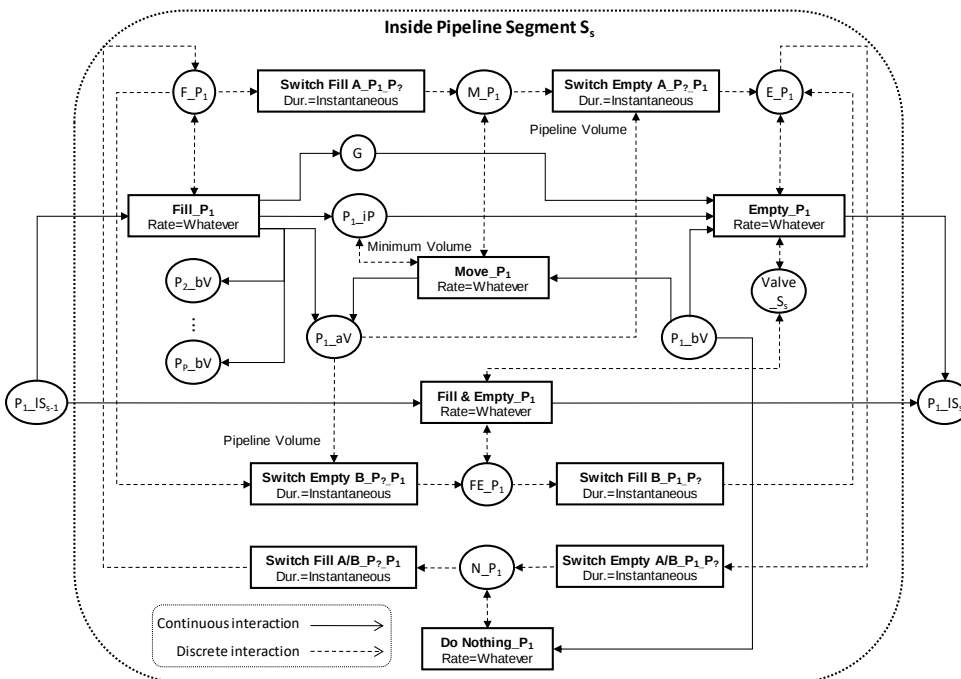
$$LC_{s,p3,t} = 7700 \quad RC_{s,p3,t} = 12100$$



Alternative product-centric approaches



- **Resource-Task Network (RTN)**
 - Larger problem size (factor 3-5)



• Generalized Disjunctive Programming (GDP)

- Disjunctions & logic propositions help derive complex constraints

$$\bigvee_p \left[\begin{array}{l} X_{s,p,t}^{S,in} \\ LC_{s,p,t} = 0 \\ f_s^{S,min} \leq F_{s,p,t}^{S,in} \leq f_s^{S,max} \\ \frac{F_{s,p,t}^{S,in}}{\rho_s^{S,max}} \leq L_t \leq \frac{F_{s,p,t}^{S,in}}{\rho_s^{S,min}} \end{array} \right] \bigvee_p \left[\begin{array}{l} X_{s,t}^{S,no i} \\ LC_{s,p,t} \leq v_s^S \quad \forall p \\ F_{s,p,t}^{S,in} = 0 \quad \forall p \\ L_t \leq h \end{array} \right] \quad \forall s, t$$

$$\bigvee_p \left[\begin{array}{l} X_{s,p,t}^{S,out} \\ RC_{s,p,t} = v_s^S \\ f_s^{S,min} \leq F_{s,p,t}^{S,out} \leq f_s^{S,max} \end{array} \right] \bigvee_p \left[\begin{array}{l} X_{s,t}^{S,no o} \\ RC_{s,p,t} \geq 0 \quad \forall p \\ F_{s,p,t}^{S,out} = 0 \quad \forall p \end{array} \right] \quad \forall s, t$$

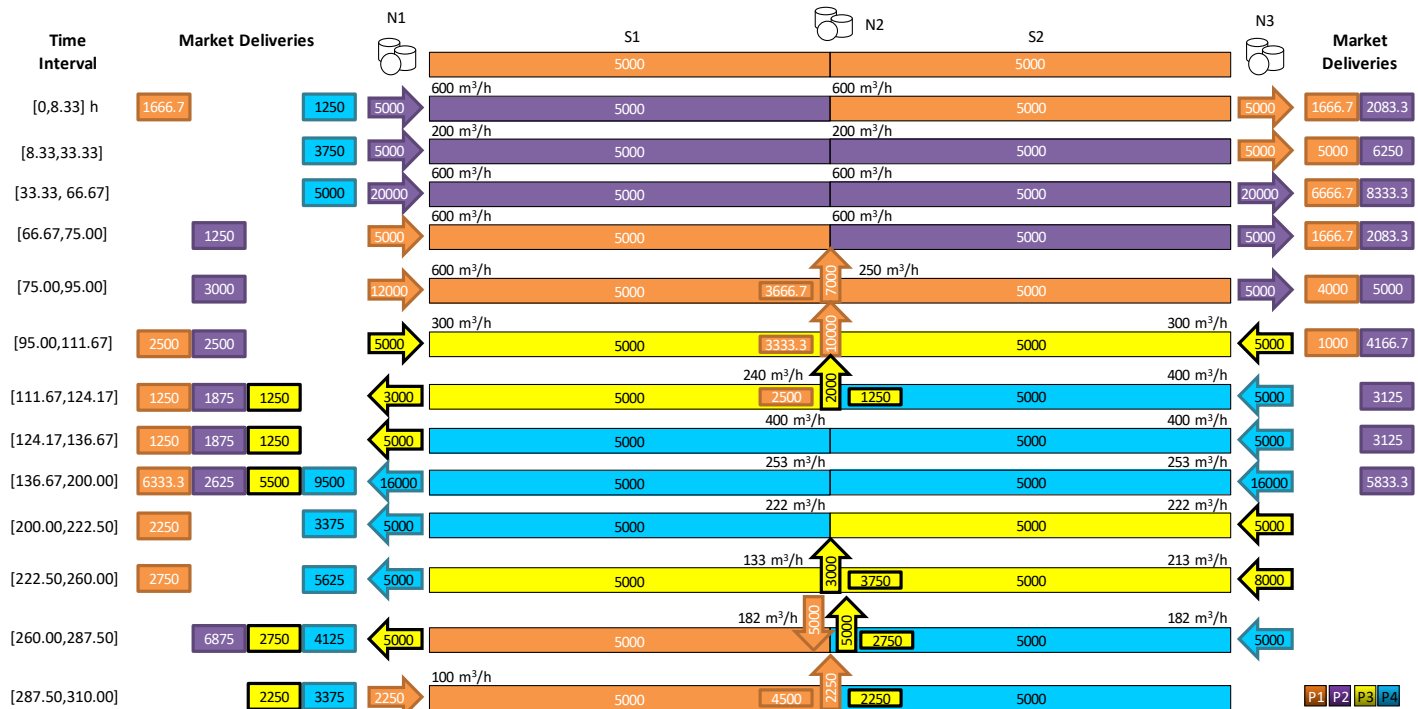
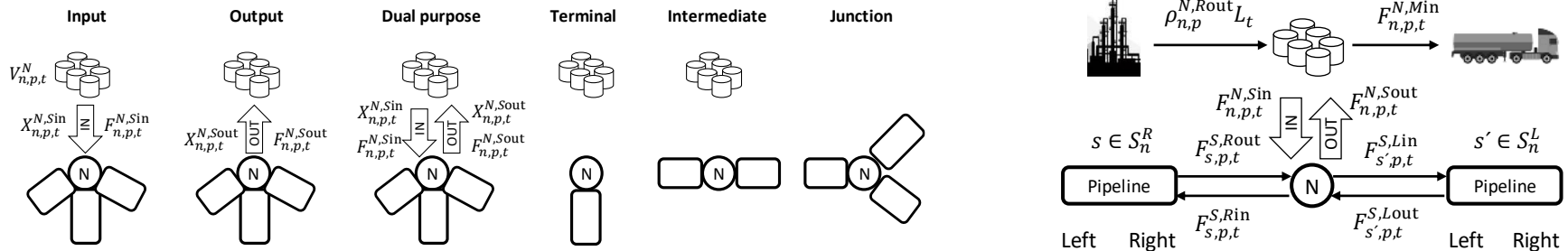
- Better computational performance

	CPU _s	RTN	GDP		RTN	GDP
Ex1		18000	7884	Ex5	16.8	1.85
Ex2		16320	111	Ex6	2.66	0.51
Ex4		110	6.22	Ex7	137	4.14

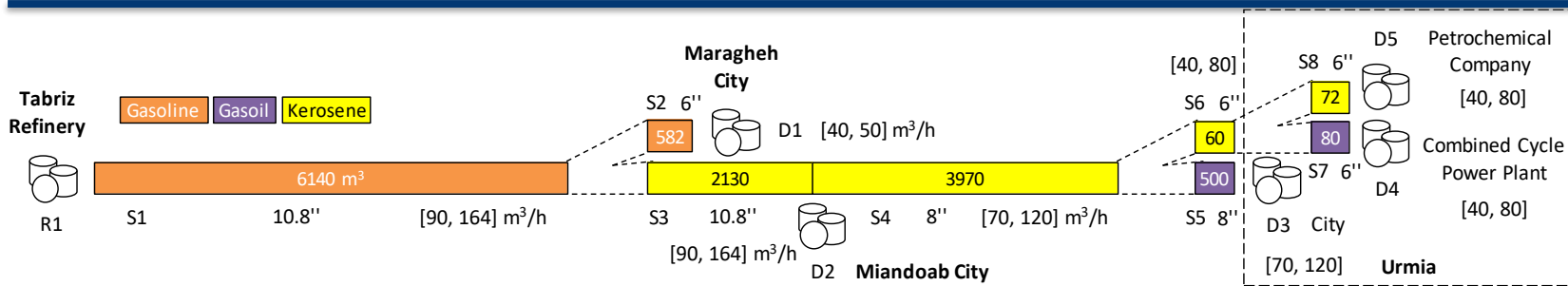
GDP-based model easily extendable



- Other configurations & systems with reversible flow



Tabriz-Urmia Pipeline in Iran



• Treelike system

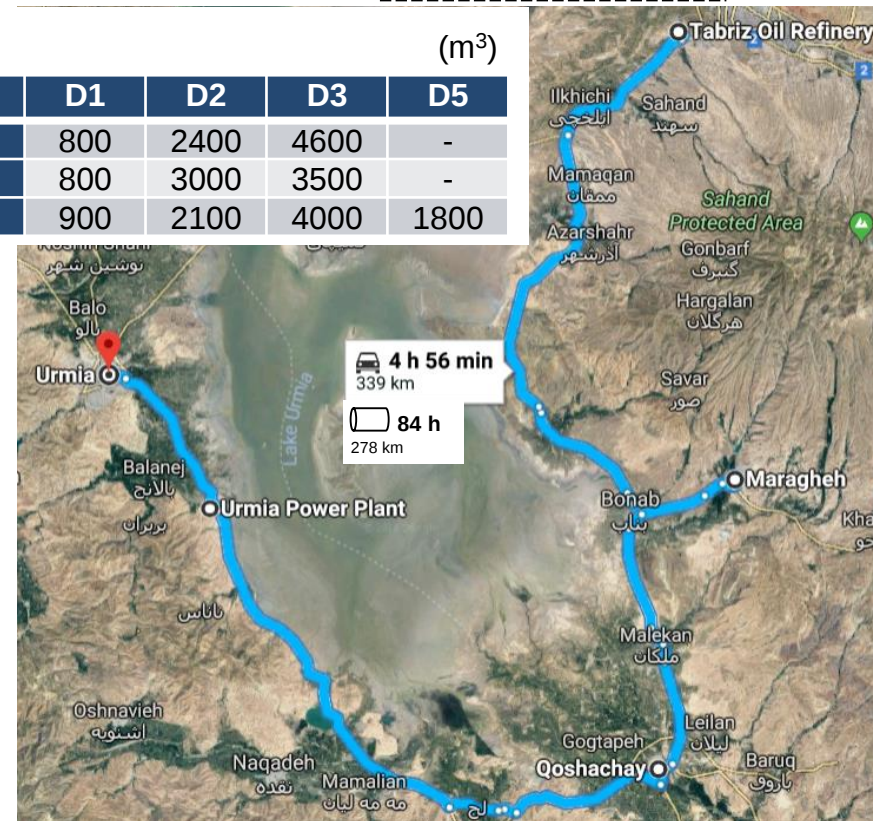
- 1 input node (R1)
- 5 output nodes (D1-D5)
- 3 products supplied

• Design parameters

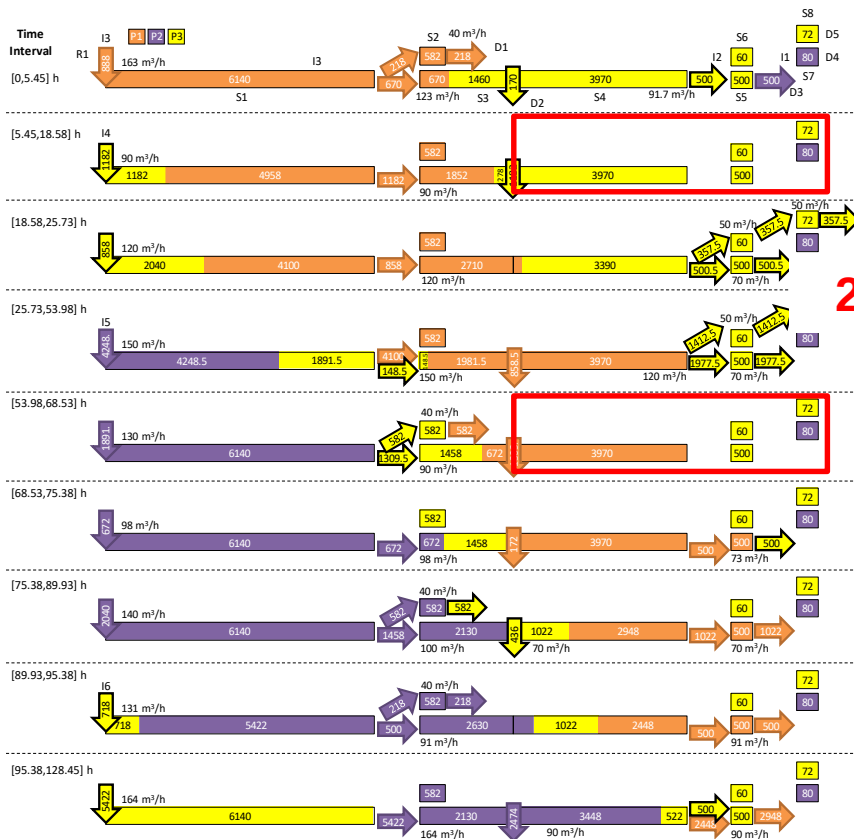
- 278 km trunk line to Urmia
 - 10.8" in first 140 km (Miandoab)
 - 8" in segment connecting to Urmia
 - Other branches have 6"
 - Operating flowrate ↓ with diameter

• Plan 1st week November 2017

	(m³)			
Weekly plan	D1	D2	D3	D5
Gasoline (P1)	800	2400	4600	-
Gasoil (P2)	800	3000	3500	-
Kerosene (P3)	900	2100	4000	1800

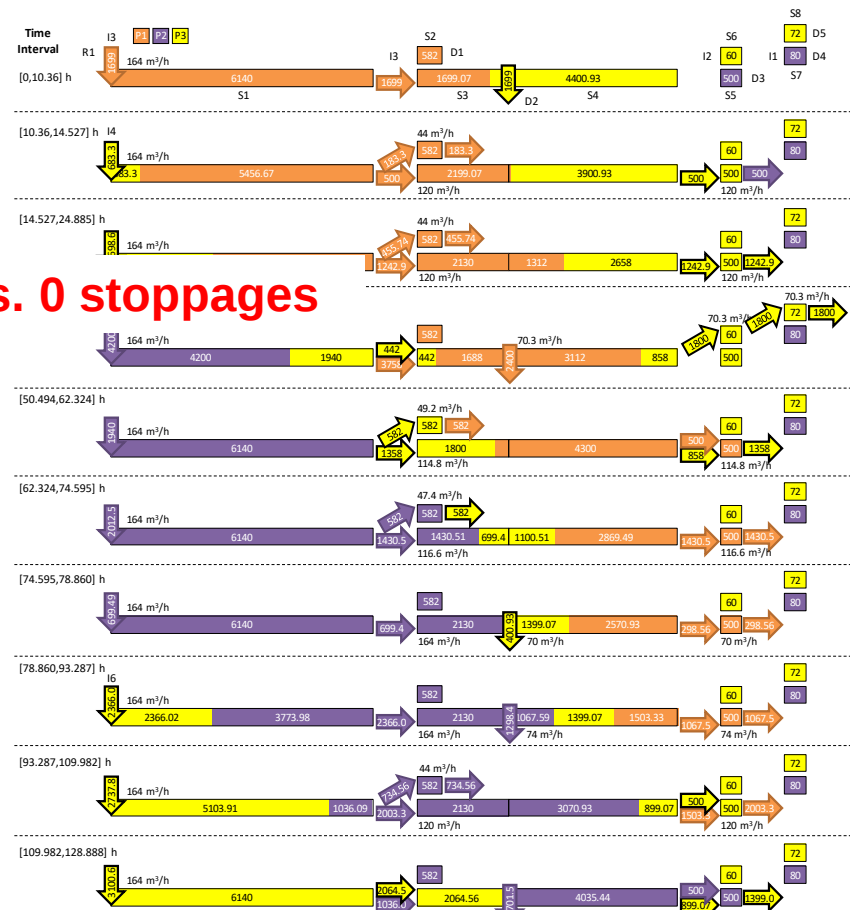


Rule-based vs. schedule from optimization



2 vs. 0 stoppages

Slightly misses weekly demand for:
P1 in D2 (60 m³)
P1 in D4 (130 m³)
P3 in D5 (30 m³)



**Meets product demand,
 while transporting 6.2% more
 in 91.5% of available time!**

PETROLEUM REFINERY

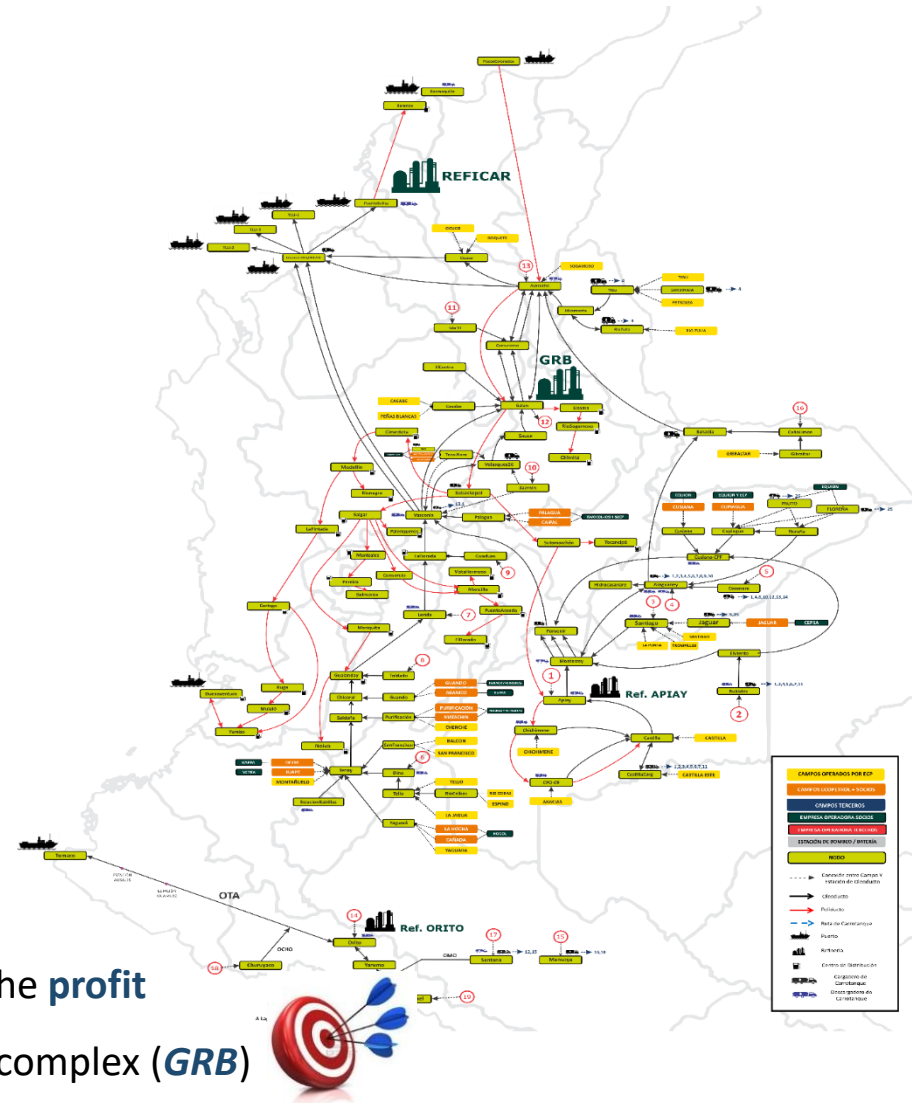
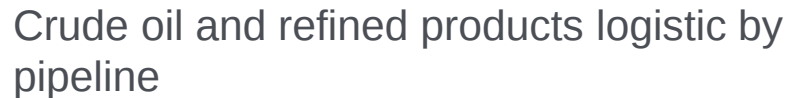
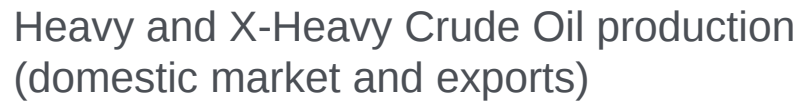
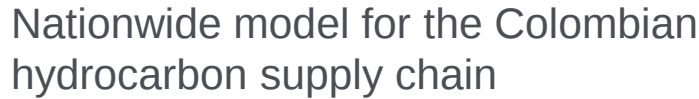


Imperial College
London

ETH zürich

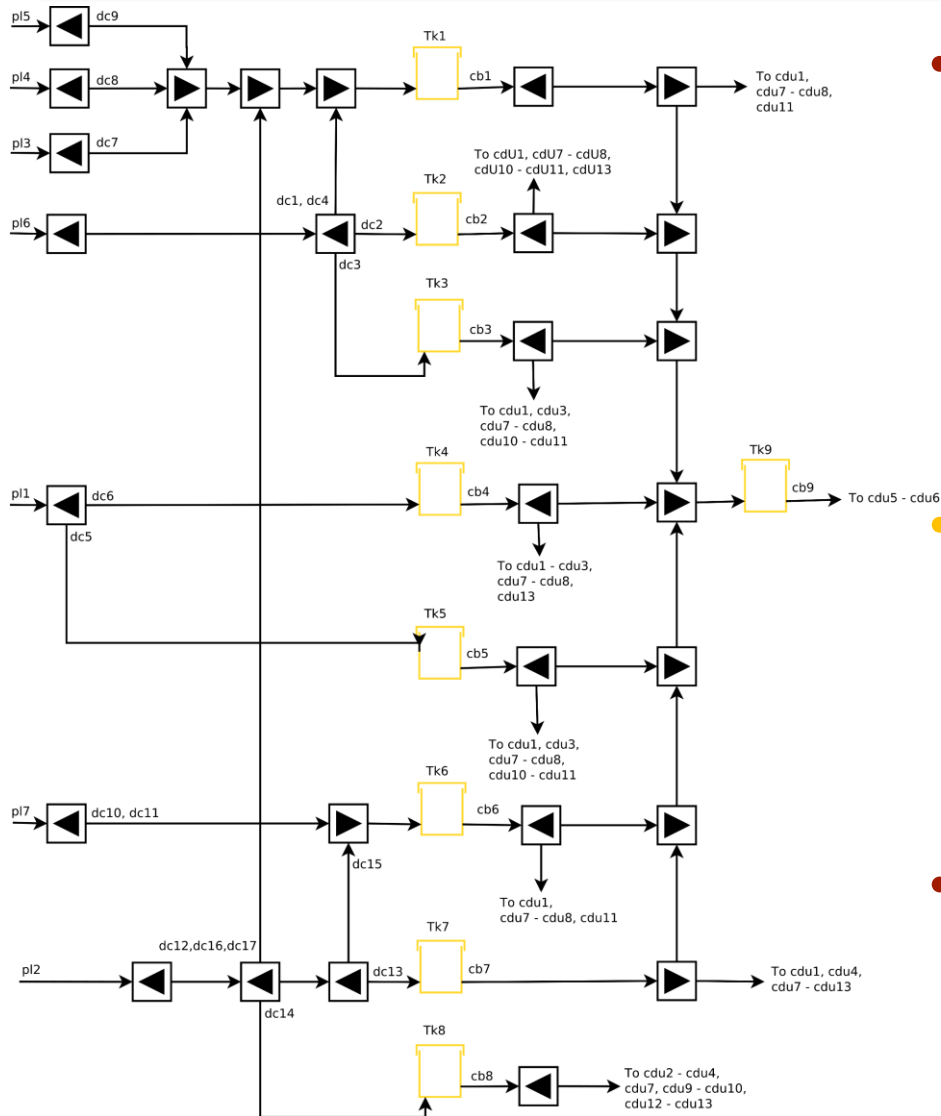


Global optimization Algorithm for large-scale
refinery planning models with bilinear terms.
[Ind. Eng. Chem. Res. 2017, 56, 530-548](#)



The **scope** is to obtain a ***near optimal solution*** for the **profit maximization** of the main refinery – petrochemical complex (**GRB**)

Crude blending network



- **Hydrocarbon market in Colombia**

- 17 types of domestic crude oil
 - 297 kbbbl/day arriving by pipeline
- 7 types of imported crude
 - Up to 15 kbbbl/day/crude

- **9 mixing tanks**

- Homogeneous crude blends with given quality properties
 - Specific gravity, Sulphur content, total acid number

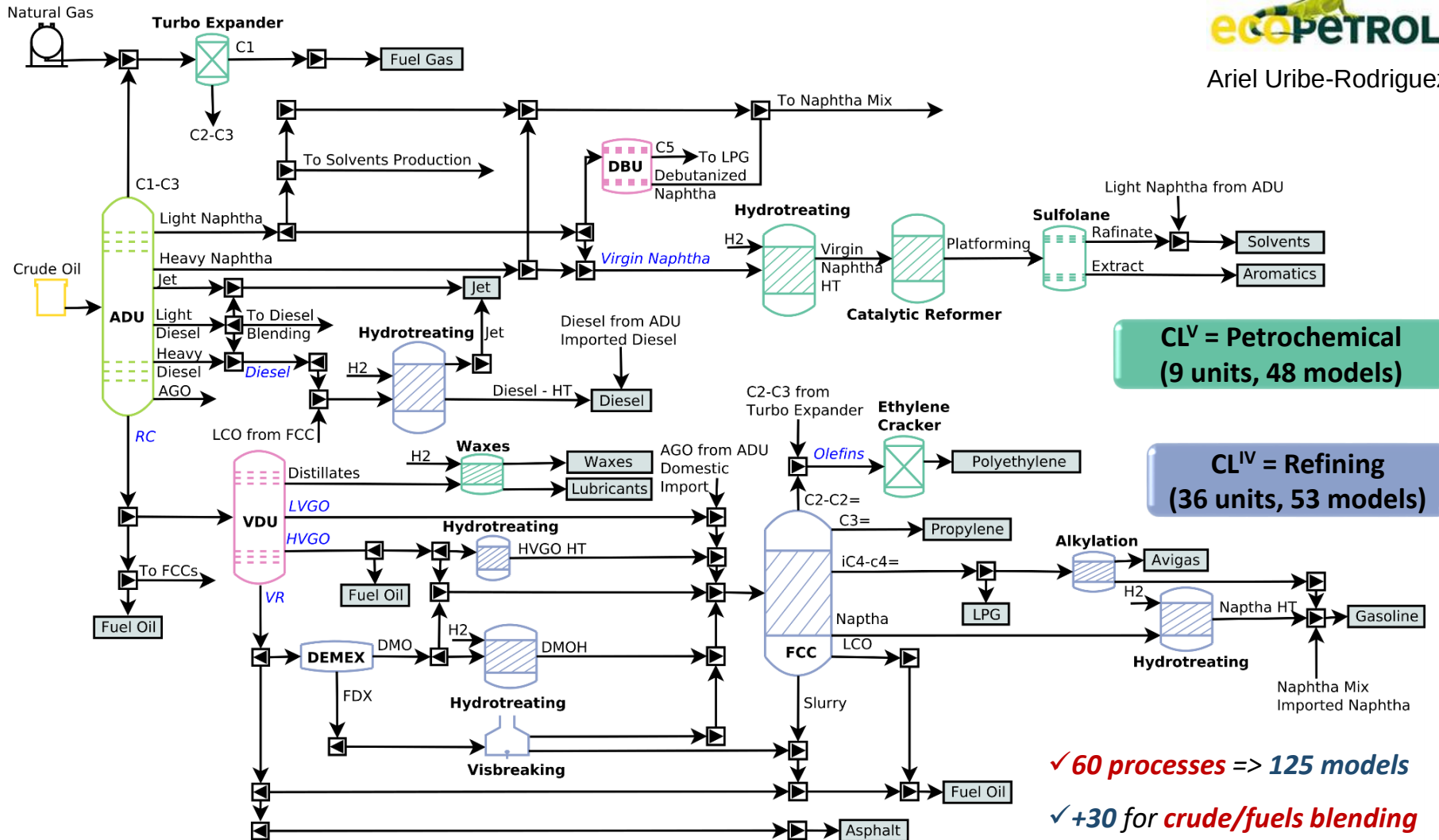
- **Crude fractionation system**

- 6 distillation columns operating in a total of 13 modes (logic units)

Integrated refinery-petrochemical complex



Ariel Uribe-Rodriguez



Additional information about the problem



- Demand for 22 grades of fuels

Fuel	LPG	Gasoline	Medium distillate	Fuel Oil	Asphalt
Streams to blend	24	23	25	10	6
Products grade	3	7	6	4	2

Fuel/Quality	Specific gravity	Sulphur content (ppm)	Cetane number	RON	RVP (kPa)	Viscosity (cp)
LPG	1					
Gasoline	1	1		1	1	
Medium distillate	1	1	1			
Fuel Oil	1	1				1
Asphalt	1	1				1

- And 26 petrochemical commodities
 - BTX, Industrial Solvents, Waxes, Propylene, Polyethylene
- Delivered by pipeline and 4 river fleet routes

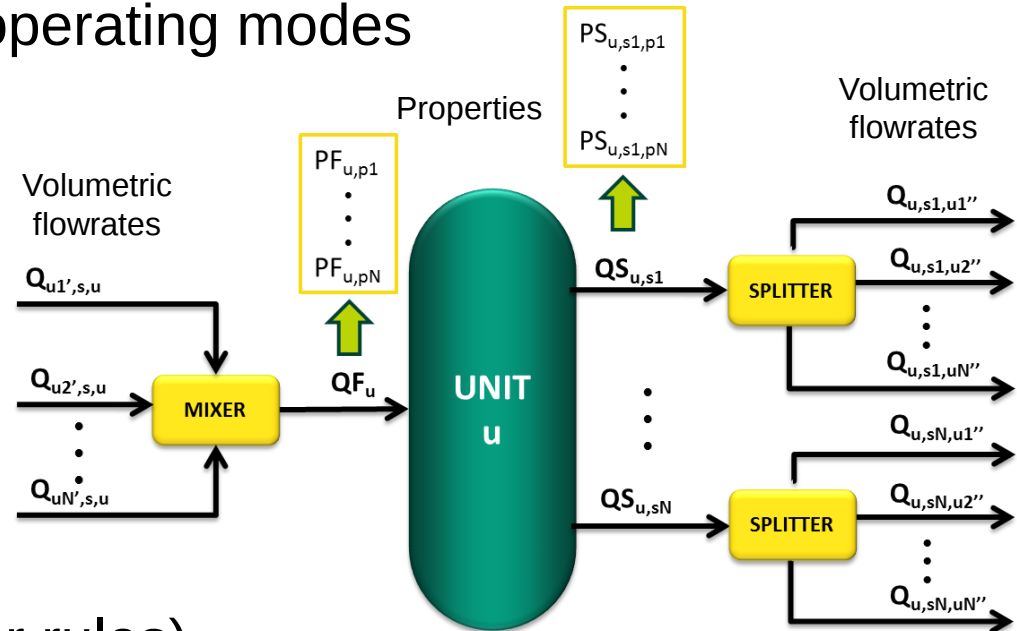
Short-term planning problem for IRPC



- **Mixed-integer nonlinear program (MINLP)**
 - Nonconvex **bilinear** and **trilinear** terms
 - Opex, pooling equations, correlations for yields & properties
 - Binary variables select operating modes
 - **Maximize profit**

- **Generic unit model**

- Volume balances
 - $QF_u = \sum_{u'} \sum_s Q_{u',s,u} \quad \forall u$
 - $QS_{u,s} = \sum_{u'} Q_{u,s,u'} \quad \forall u, s$
- Property balances (linear rules)
 - Volume basis: $QF_u PF_{u,p} = \sum_{u'} \sum_s Q_{u',s,u} PS_{u',s,p} \quad \forall u, p$
 - Weight: $QF_u PF_{u,p} PF_{u,SPG} = \sum_{u' \in \epsilon} \sum_s Q_{u',s,u} PS_{u',s,p} PS_{u',s,SPG} \quad \forall u, p$



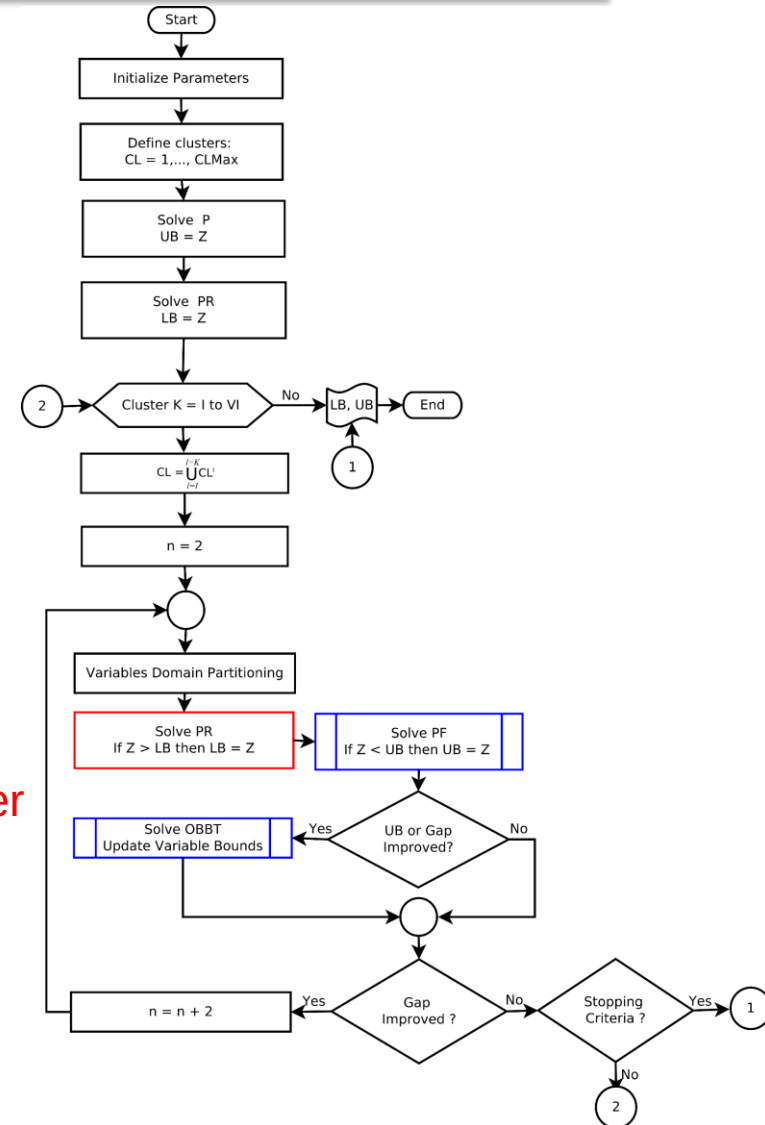
Deterministic global optimization algorithm



- Process units divided into 6 clusters
 - Similar functionality, follows workflow

Cluster	Process Units	Physical Units	Logical Units
CL^I	Logistic & Crude Allocation	8	8
CL^{II}	Crude Distillation	6	13
CL^{III}	Vacuum & Debutanizer Columns	8	11
CL^{IV}	Refining	36	53
CL^V	Petrochemical	9	48
CL^{VI}	Fuels Blending	22	22

- 2-stage MILP-NLP decomposition
 - MILP relaxation **PR** of MINLP problem **P**
 - McCormick ('76) for units outside active cluster
 - Piecewise McCormick (Bergamini et al. '05) for cluster units
 - Optimal solution as # partitions $n \rightarrow \infty$
 - Optimality-based bound tightening **OBBT**
 - Reduces variables domain & optimality gap



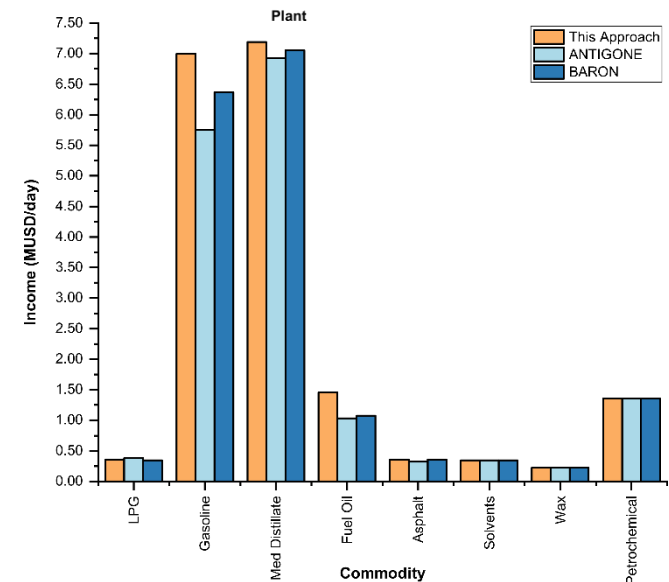
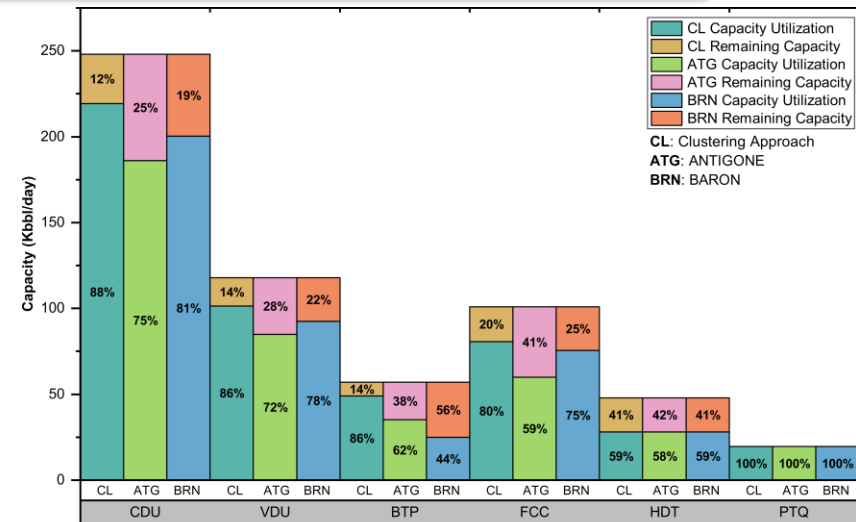
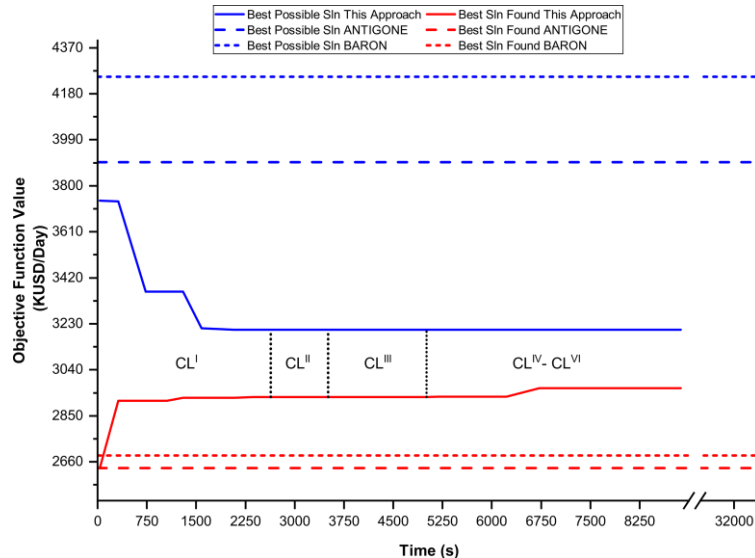
Comparison to commercial global solvers



• Significantly higher profit!

Algorithm	Cluster	ANTIGONE	BARON
Profit [k\$/day]	2964	2634 (-12%)	2687 (-10%)
Optimality Gap	8%	48%	58%
Runtime [h]	5.7	10	10

– Very large problem, clusters needed to improve quality & optimality gap



• IRPC capacity = 219 kbbl/day (crude)

CRUDE-OIL BLENDING



Global optimal scheduling of crude oil blending operations with RTN continuous-time and multiparametric disaggregation.

[Ind. Eng. Chem. Res. 2014, 53, 15127-15145](#)

Source-based discrete and continuous-time formulations for the crude oil pooling problem.

[CACE 2016, 93, 382-401](#)



Crude oil blend scheduling



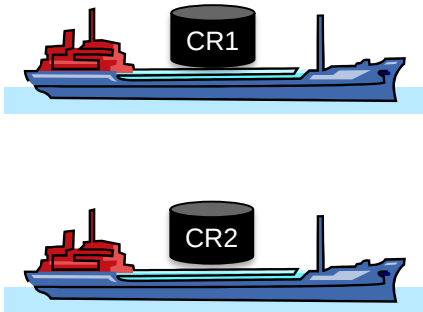
- Marine vessels unload sequentially following their order of arrival at_{mv}

$t = 0$

$t = at_{mv_1}$

$t = at_{mv_2}$

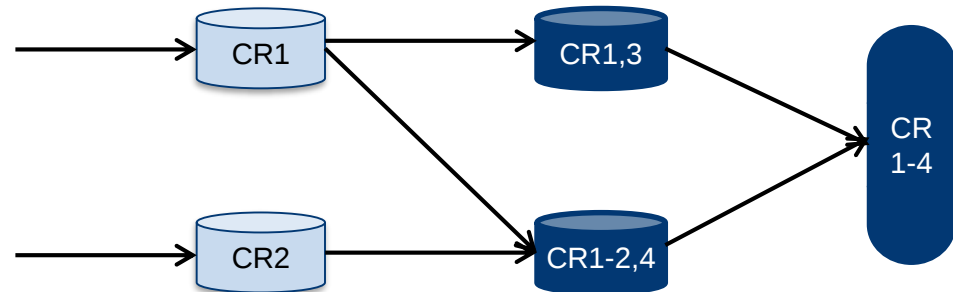
Marine vessels (MV)



Storage tanks (ST)

Charging tanks (CT)

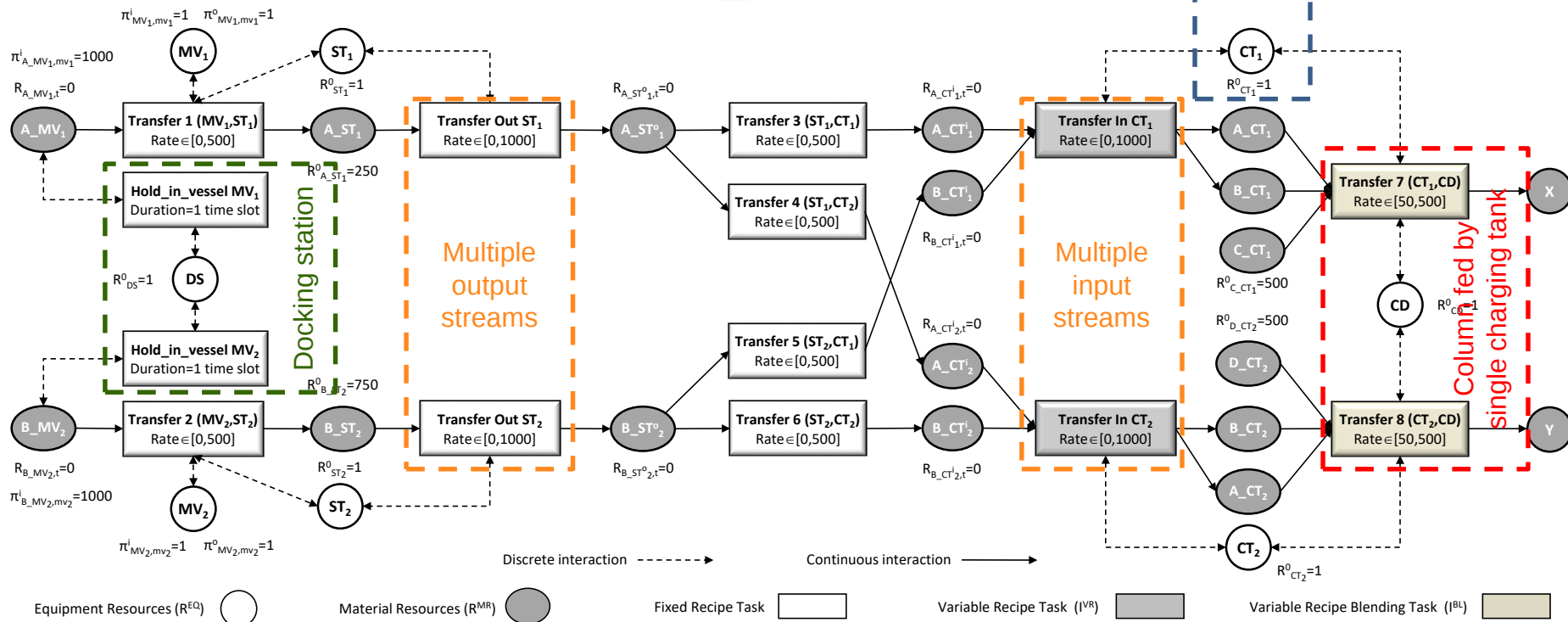
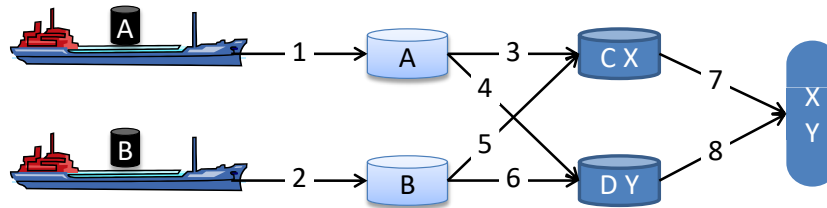
Distillation units (CD)



- Logistic constraints (within a time slot)
 - Either flow-in or out in tanks handling different crudes (blending tanks)
 - A charging tank feeds at most one CDU
 - A CDU receives crude from a single charging tank
- Mix properties computed assuming linear blending, $\in [c_{tk,pr}^{min}, c_{tk,pr}^{max}]$
- Crude demand from charging tanks $\in [d_{ct}^{min}, d_{ct}^{max}]$

RTN process model

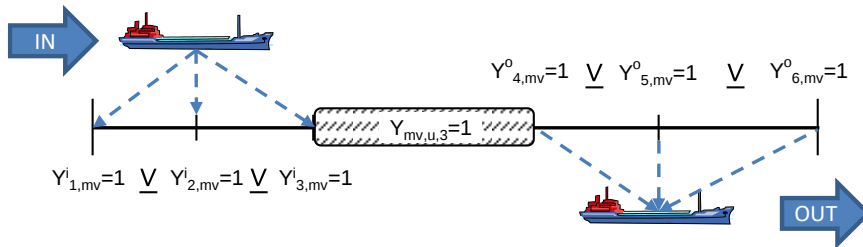
- Need for additional transfer tasks



GDP model with logic propositions



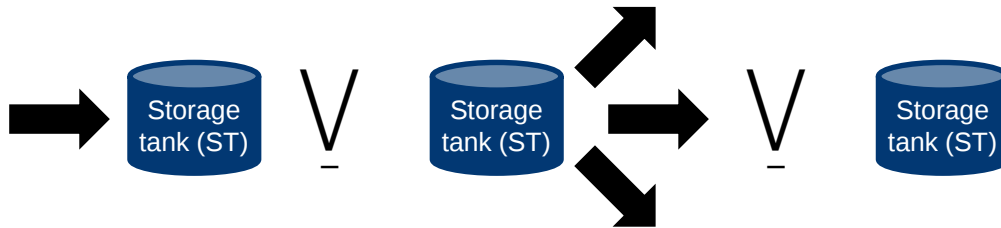
- Transfer from marine vessel to storage tank



$$Y_{mv,u,t} \Rightarrow \left(\bigvee_{t' \leq t, t' \in T_{mv}^i} Y_{t',mv}^i \wedge \bigvee_{t' > t, t' \in T_{mv}^o} Y_{t',mv}^o \right) \forall u, mv, t$$

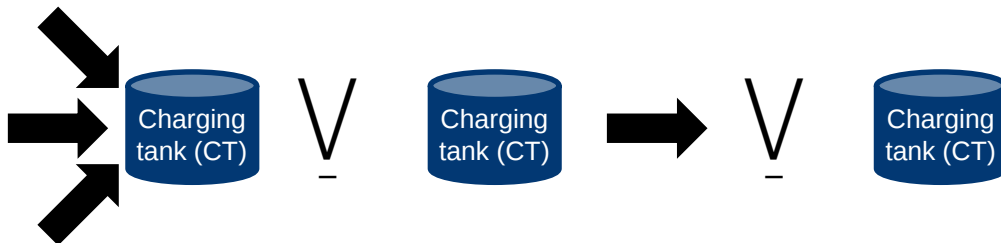
$$\bigvee_{mv} \bigvee_u Y_{mv,u,t} \bigvee Y_t^{no\ mv} \forall t$$

- Single docking station
- Multiple output streams in storage tanks



$$Y_{mv^*,u,t} \bigvee Y_{u,u',t} \bigvee Y_{u,t}^{no\ io} \forall u \in B \cap ST, u', t$$

- Charging tanks with single active output stream



$$Y_{u',u,t} \bigvee_{u''} Y_{u,u'',t} \bigvee Y_{u,t}^{no\ io} \forall u \in B \cap CT, u', t$$

Discrete vs. continuous-time



• Advantages of discrete-time

- Simpler model
- Tighter MILP-LP relaxation
- Easier to account for time-varying inventory costs
 - Better for **cost minimization**

	Discrete-time		Continuous-time	
Problem	Slots $ T $	Solution (k\$)	Solution (k\$)	Slots $ T $
P1	97	209.585	210.537	8
P2	81	319.140	320.496	8
P3	97	284.781	287.000	8
P4	121	319.875	333.331	7

• Advantages of continuous-time

- More accurate model
- Fewer slots to represent schedule
- ↓ nonlinear blending constraints
 - Better for **gross margin maximization**

	Discrete-time		Continuous-time	
	Slots $ T $	Solution (k\$)	Solution (k\$)	Slots $ T $
	97	7983	7985	4
	81	10240	10246	7
	49	8542	8574	8
	121	13258	13258	7

• Major surprise!

- **Zero MINLP-MILP gap** from McCormick envelopes for bilinear terms!
 - Better than BARON & GloMIQO

Approach		Cost [\$]	Gap	CPUs		Cost [\$]	Gap	CPUs
McCormick	P1	209585	0.0000%	72.6	P3	284781	0.0000%	346
GloMIQO		209585	0.0001%	1557		284781	11.1%	3600
BARON		209585	0.0001%	305		397208	112%	3600
McCormick	P2	319140	0.0000%	662	P4	322300	7.6%	3600
GloMIQO		319252	10.9%	3600		No sol.	17.6%	3600
BARON		319140	38.5%	3600		324746	37.9%	3600

FINAL SLIDE!

- Optimization-based decision making can significantly improve Key Performance Indicators
- Scheduling is not only about equipment availability
 - Real-life problems feature other challenging constraints
 - Time dependent utility cost and/or availability,
 - Logistics, blending
 - RTN & GDP are powerful modeling tools
 - Both need some time to master! 😊
- Despite progress over the last 30 years, still hard to predict best approach for a particular problem
 - No two problems are the same
 - Time representation is critical
- Increased emphasis on nonlinear models (MINLP)